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Sec: 11 (12:50-1:40) 14 (1:50-2:40)

MATH-102

Term-141

Class-QUIZ-on-Series

(show all your work and circle one letter to get a full mark or you will get zero)

1) The series $\sum_{n=1}^{\infty} \frac{(-n!)^4}{(4n+3)!}$ is

- a) conditionally convergent
- b) a divergent p series
- c) divergent by the ratio test
- d) a series for which the ratio test is inconclusive
- e) absolutely convergent
- f) none of the above

$|a_n| = \frac{(n!)^4}{(4n+3)!}$
 $|a_{n+1}| = \frac{((n+1)!)^4}{(4n+7)!}$
 $\frac{|a_{n+1}|}{|a_n|} = \frac{((n+1)!)^4}{(4n+7)!} \cdot \frac{(4n+3)!}{(n!)^4}$
 $= \frac{(n+1)^4}{(4n+7)(4n+6)(4n+5)(4n+4)}$

4) The series $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{n+1}+2)\sqrt{n}}$ is $\approx \sum \frac{1}{\sqrt{n}\sqrt{n}} \approx \frac{1}{n}$

- a) diverges by the limit comparison test
- b) convergent by the integral test
- c) convergent by the ratio test
- d) convergent by the root test
- e) divergent by the ratio test
- f) none of the above

$\lim_{n \rightarrow \infty} \frac{1/n}{1/((\sqrt{n+1}+2)\sqrt{n})} = 1$
 $\therefore \lim_{n \rightarrow \infty} \frac{(\sqrt{n+1}+2)\sqrt{n}}{n} = 1$ but diverg

2) The series $\sum_{n=1}^{\infty} \frac{3^n n^n}{2^{2n+1}}$ is

- a) diverges by the root test
- b) a convergent p series
- c) converges by the root test
- d) a series for which the root test is inconclusive
- e) a divergent geometric series
- f) none of the above

$(a_n)^{1/n} = \frac{3n}{2^{2+1/n}}$
 $\lim_{n \rightarrow \infty} (a_n)^{1/n} = \infty$

5) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^{n+1} + (n+2)^3}$ is

- a) conditionally convergent
- b) divergent
- c) absolutely convergent
- d) convergent by the integral test
- e) divergent by the alternating series test
- f) none of the above

$|a_n| = \frac{1}{4^{n+1} + (n+2)^3}$
 $(n+2)^3 + 4^{n+1} > 4^{n+1}$
 $\frac{1}{(n+2)^3 + 4^{n+1}} < \frac{1}{4^{n+1}}$
 by comparison test $\sum |a_n|$ converg

3) The series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+3} - \sqrt{n+2})$ is

- a) diverges by the limit comparison test
- b) conditionally convergent
- c) absolutely convergent
- d) diverges by the divergent test
- e) divergent by the ratio test
- f) none of the above

Study $\sum |a_n|$
 $= \sum (\sqrt{n+3} - \sqrt{n+2})$
 multiply by conjugate
 $= \sum \frac{1}{\sqrt{n+3} + \sqrt{n+2}}$ similar
 to $\sum \frac{1}{\sqrt{n}}$
 $\lim_{n \rightarrow \infty} \frac{1/\sqrt{n}}{1/(\sqrt{n+3} + \sqrt{n+2})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} + \sqrt{n+2}}{\sqrt{n}} = 2$

3) The series $\sum_{n=1}^{\infty} (3\sqrt{3} - \sqrt[3]{3})^{\frac{n}{2}}$ is

- a) the root test is inconclusive
- b) conditionally convergent
- c) a divergent geometric series
- d) convergent by the root test
- e) divergent by the root test
- f) none of the above

$(a_n)^{1/n} = (3\sqrt{3} - \sqrt[3]{3})^{\frac{1}{2}}$
 $\lim (a_n)^{1/n} = (3\sqrt{3} - \sqrt[3]{3})^{\frac{1}{2}}$
 $= \sqrt{3\sqrt{3}} > 1$
 diverg by root test

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7) If the sum of the first n terms of a series $\sum_{n=1}^{\infty} a_n$ is given by

$$S_n = \frac{2n}{n+1} \quad \text{then} \quad a_{10} =$$

- a) 1/110
- b) 4/110
- c) 2/120
- d) 2/101
- e) 2/101

f) none of the above

$$a_n = S_n - S_{n-1}$$

$$\begin{aligned} a_{10} &= S_{10} - S_9 \\ &= \frac{20}{11} - \frac{18}{10} \\ &= \frac{200 - 198}{110} = \frac{2}{110} \end{aligned}$$

8) The series $\sum_{n=2}^{\infty} \frac{6}{n^2-1} = 3 \sum_{n=2}^{\infty} \frac{2}{n^2-1} = 3 \left[\sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) \right]$

a) 9/2

- b) 3
- c) 3/2
- d) 0
- e) 6

f) none of the above

$$\begin{aligned} &= 3 \left\{ \sum_{n=2}^{\infty} \frac{1}{n-1} - \frac{1}{n} + \sum_{n=2}^{\infty} \frac{1}{n} - \frac{1}{n+1} \right\} \\ &= 3 \left[1 - 0 + \frac{1}{2} - 0 \right] \\ &= 3 \left[\frac{3}{2} \right] = \frac{9}{2} \end{aligned}$$

10) The minimum number of terms needed to estimate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+3)^4}$ within 0.0001 (error) is

a) $n=5$

b) $n=7$

c) $n=10$

d) $n=13$

e) $n=6$

f) none of the above

$$U_n = \frac{1}{(2n+3)^4}$$

$$U_{n+1} \leq \frac{1}{10000}$$

$$\frac{1}{(2n+5)^4} \leq \frac{1}{10000}$$

$$(2n+5)^4 \geq 10000$$

$$2n+5 \geq 10$$

$$2n \geq 5 \Rightarrow n \geq 2.5$$

$$n \geq 3$$

The smallest choice is $n=5$

9) If $\{S_n\}$ is the sequence of partial sums of the series

$$\sum_{n=3}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right) \quad \text{then} \quad S_n =$$

a) $\frac{1}{3} - \frac{1}{n+1} + \frac{1}{4} - \frac{1}{n+2}$

b) $2 - \frac{1}{n+1} - \frac{1}{n+2}$

c) $\frac{1}{3} - \frac{1}{n} + \frac{1}{4} - \frac{1}{n+2}$

d) $\frac{1}{6} - \frac{1}{n+1} - \frac{1}{n+2}$

e) $\frac{1}{3} - \frac{1}{n} + \frac{1}{4} - \frac{1}{n+1}$

f) none of the above

$$\sum_{n=3}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right) = \sum_{n=3}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) + \sum_{n=3}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$\text{Hence } S_n = \left(\frac{1}{3} - \frac{1}{n+1} \right) + \left(\frac{1}{4} - \frac{1}{n+2} \right)$$