

KEY

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1) Determine whether the integral is convergent or divergent. If it is convergent, find its value.

$$I = \int_0^{\infty} \frac{e^x}{e^{2x} + 3} dx$$

$u = e^x \rightarrow du = e^x dx$

$$I = \int_1^{\infty} \frac{du}{u^2 + 3} = \lim_{t \rightarrow \infty} \int_1^t \frac{du}{u^2 + 3} \quad (4)$$

$$= \lim_{t \rightarrow \infty} \left[ \frac{1}{\sqrt{3}} \tan^{-1} \frac{u}{\sqrt{3}} \right]_1^t$$

$$= \lim_{t \rightarrow \infty} \left[ \frac{1}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} \right]$$

$$= \frac{1}{\sqrt{3}} \lim_{t \rightarrow \infty} \left[ \tan^{-1} \frac{t}{\sqrt{3}} - \tan^{-1} \frac{1}{\sqrt{3}} \right]$$

$$= \frac{1}{\sqrt{3}} \left[ \frac{\pi}{2} - \tan^{-1} \frac{1}{\sqrt{3}} \right] \quad (5)$$

So, the integral is convergent and its value is  $\frac{1}{\sqrt{3}} \left[ \frac{\pi}{2} - \tan^{-1} \frac{1}{\sqrt{3}} \right]$

2) Match the integral with the appropriate method

$\int e^x \cos^2 x dx$	g	2
$\int \frac{x+2}{(x-1)(x^2+1)} dx$	d	2
$\int x \csc^2 x \cot x dx$	c	2
$\int x(x+1)e^x dx$	f	2
$\int \sin(\ln x^2) dx$	a	2
$\int_4^6 (x^2+1) \operatorname{sech}(\ln x) dx$	e	2

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| a) two integration by part then back to original                     |
| b) By part with $dv = \csc^2 x$                                      |
| c) By part with $dv = \csc^2 x \cot x$                               |
| d) Partial fraction  |
| e) Simplify the integrand to become $2x$                             |
| f) Integration by part several times                                 |
| g) Trig. Identity then two integration by part then back to original |
| h) Trig substitution with $u = \cot x$                               |
| i) By part with $u = \operatorname{sech}(x)$                         |