

(show all your work and circle one letter to get a full mark or you will get zero)

1) The curve  $y = \sqrt{4x - x^2}$ ,  $2 \leq x \leq 6$  is revolved about the x-axis. The area of the generated surface is equal to

- (a)  $12\pi$
- (b)  $16\pi$**
- (c)  $4\pi$
- (d)  $4\pi + 16$
- (e)  $4\pi + 2$
- (f) none of the above

$$y' = \frac{1}{2}(4x - x^2)^{-1/2} (4 - 2x)$$

$$= \frac{(2-x)}{\sqrt{4x-x^2}}$$

$$\sqrt{1+y'^2} = \sqrt{1 + \frac{(2-x)^2}{4x-x^2}} = \sqrt{\frac{4x-x^2 + x^2 - 4x + 4}{4x-x^2}}$$

$$= \sqrt{\frac{4}{4x-x^2}} = \frac{2}{\sqrt{4x-x^2}}$$

$$S = \int_a^b 2\pi y \sqrt{1+y'^2} dx = \int_2^6 2\pi \sqrt{4x-x^2} \left[ \frac{2}{\sqrt{4x-x^2}} \right] dx$$

$$= \int_2^6 4\pi dx = 4\pi \int_2^6 dx = 4\pi [6-2] = 16\pi$$

2) The length of the curve  $x = (1 - y^{2/3})^{3/2}$ ,  $0.5 \leq y \leq 1$

- (a)  $\frac{2}{3} - \sqrt{2}$
- (b) 6
- (c)  $3\sqrt[3]{4} - 1$
- (d)  $\frac{3}{2} - \frac{3}{2\sqrt[3]{4}}$**
- (e)  $3\sqrt[3]{4} + 1$
- (f) none of the above

$$\frac{dx}{dy} = \frac{3}{2} (1 - y^{2/3})^{1/2} \cdot \left( \frac{2}{3} y^{-1/3} \right)$$

$$= y^{-1/3} (1 - y^{2/3})^{1/2}$$

$$\sqrt{1+x'^2} = \sqrt{1 + y^{-2/3} (1 - y^{2/3})} = \sqrt{1 + y^{-2/3} - 1}$$

$$= \sqrt{y^{-2/3}} = y^{-1/3}$$

$$\int_{1/2}^1 y^{-1/3} dy = \left[ \frac{3}{2} y^{2/3} \right]_{1/2}^1 = \frac{3}{2} \left[ 1 - \left( \frac{1}{2} \right)^{2/3} \right]$$

$$= \frac{3}{2} \left[ 1 - \frac{1}{\sqrt[3]{4}} \right] = \frac{3}{2} - \frac{3}{2\sqrt[3]{4}}$$