

1) Using the method of cylindrical shells, set up (but DO NOT EVALUATE) an integral for the volume of the solid generated by revolving

(a) The region enclosed by the curves  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 3$  about the  $y$ -axis [ sketch the region and a typical rectangle]

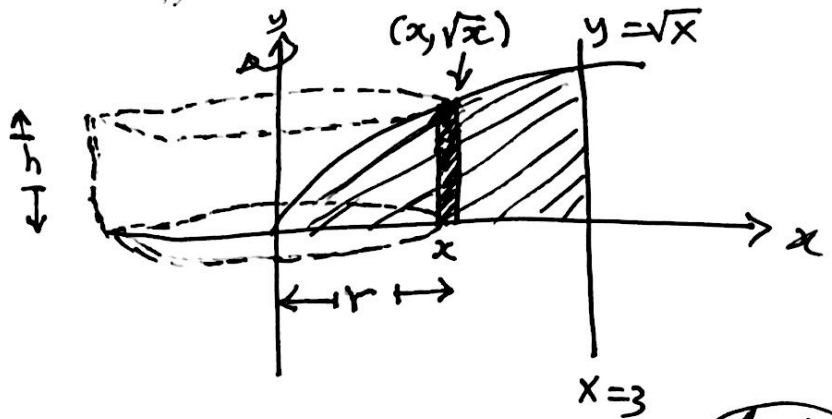
$$r = x$$

$$h = \sqrt{x}$$

$$V = \int_a^b 2\pi r h$$

$$= \int_0^3 2\pi (x)(\sqrt{x}) dx$$

(2pts) (2pts) (2pts)



(4pts)

(b) Find the value(s) of  $c$  which satisfy the mean value theorem for integrals where  $f(x) = \sin x$

over the interval  $[\frac{\pi}{6}, \frac{\pi}{2}]$

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{\frac{\pi}{2} - \frac{\pi}{6}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x dx = \frac{1}{\frac{3\pi - \pi}{6}} \left[ -\cos x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{1}{\frac{\pi}{3}} \left[ -\cos \frac{\pi}{2} + \cos \frac{\pi}{6} \right] = \frac{3}{\pi} \cos \frac{\pi}{6} = \frac{3}{\pi} \frac{\sqrt{3}}{2}$$

by mean value thm for integrals there exists  $c$  such that

$$f(c) = f_{avg} \Rightarrow \sin c = \frac{3\sqrt{3}}{2\pi} \Rightarrow c = \sin^{-1} \left( \frac{3\sqrt{3}}{2\pi} \right)$$