

$$1) \int x \ln(x) dx = I$$

integration by part

$$u = \ln x \quad dv = x dx \quad (4)$$

$$du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$$

$$I = \frac{1}{2} x^2 \ln x - \int \left(\frac{1}{2} x^2\right) \left(\frac{1}{x}\right) dx \quad (3)$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \left(\frac{1}{2} x^2\right) + C$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C \quad (3)$$

$$5) \int \tan^3 x \sec^3 x dx = I$$

$$I = \int \tan^2 x \sec^2 x \sec x \tan x dx \quad (2)$$

$$= \int (\sec^2 x - 1) \sec^2 x \cdot \sec x \tan x dx \quad (2)$$

$$\text{Let } u = \sec x, \quad du = \sec x \tan x dx \quad (2)$$

$$= \int (u^2 - 1) u^2 du = \int (u^4 - u^2) du$$

$$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C \quad (2)$$

$$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C \quad (2)$$