Name: KEY **ID:** KEY Sec8(9:00-9:50) Sec9(10:00-10:50)

MATH-102 Term-152 Q2

Show all your work and circle one answer

1) Let $F(x)=\int_{\sin x}^{\cos(3x)} \frac{5dt}{\sqrt{1+4t^2}}$ then $F'(\frac{\pi}{2})=$

(a) 15
(b) 16
(c) -14
(d) 3
(e) 10
(f) none of the above

By Fund. Thm of Calculus

$$F'(x)=\frac{5(-\sin 3x)(3)}{\sqrt{1+4\cos^2(3x)}}-\frac{5(\cos x)}{\sqrt{1+4\sin^2 x}}$$

$$F'(\tfrac{\pi}{2})=\frac{-15\sin\frac{3\pi}{2}}{\sqrt{1+4\cos^2(\frac{3\pi}{2})}}-\frac{5\cos\frac{\pi}{2}}{\sqrt{1+4\sin^2\frac{\pi}{2}}}$$

$$=\frac{(-15)(-1)}{\sqrt{1+0}}-0$$

$$=15$$

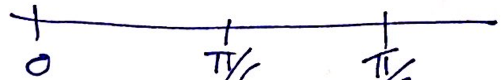
2) If the velocity of a particle moving along a straight line is given by $v(t)=\frac{1}{2}-\sin t$ the distance traveled during the interval time $[0,\frac{\pi}{2}]$

(a) $\sqrt{3}-1-\pi/12$
(b) 0
(c) 1
(d) $4-\pi$
(e) 12
(f) none of the above

$v(t)=0 \Rightarrow \frac{1}{2}-\sin t=0$

$\sin t=\frac{1}{2}$

$t=\pi/6$



$$\text{total distance}=\int_0^{\pi/2}|v(t)|\,dt$$

$$|v(t)|=\begin{cases}\frac{1}{2}-\sin t & t\in[0,\pi/6]\\ -(\frac{1}{2}-\sin t) & t\in[\pi/6,\pi/2]\end{cases}$$

$$\int_0^{\pi/2}|v(t)|\,dt=\int_0^{\pi/6}(\tfrac{1}{2}-\sin t)\,dt+\int_{\pi/6}^{\pi/2}-(\tfrac{1}{2}-\sin t)\,dt$$

$$=\left[\tfrac{1}{2}t+\cos t\right]_0^{\pi/6}-\left[\tfrac{1}{2}t+\cos t\right]_{\pi/6}^{\pi/2}$$

$$=\left(\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1\right)-\left(\frac{\pi}{4}-\frac{\pi}{12}-\frac{\sqrt{3}}{2}\right)$$

$$=\sqrt{3}-1-\pi/12$$