| Name: | ID: | Sec: 10 |
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MATH-102 Term-142 CQ-17 **FORM A**

**(show all your work and circle one letter to get a full mark or you will get zero)**

1) The first three nonzero terms of the Taylor series $f(x)=\sqrt{x}$ about $a=1$ are given by

(a) $1+\frac{1}{2}(x-1)-\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3$

(b) $1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3$

(c) $1+\frac{1}{2}(x-1)-\frac{1}{4}(x-1)^2+\frac{5}{8}(x-1)^3$

(d) $1+\frac{1}{2}(x-1)+(x-1)^2+\frac{3}{8}(x-1)^3$

(e) $1+(x-1)-\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3$

(f) none of the above

$f=x^{1/2}$, $f(1)=1$

$f'=\frac{1}{2}x^{-1/2}$, $f'(1)=\frac{1}{2}$

$f''=-\frac{1}{4}x^{-3/2}$, $f''(1)=-1/4$

$f'''=\frac{3}{8}x^{-5/2}$, $f'''(1)=\frac{3}{8}$

coeff

$(1)$

$\frac{1}{2}(x-1)$

$-\frac{1}{4(2!)}(x-1)^2$

2) Using the binomial series, we have, $\sqrt{1+8x^3}=$

(a) $1+4x^3-8x^6+32x^9$

(b) $\frac{1}{2}+8x^3-4x^6+16x^9$

(c) $1+4x^3+4x^6+32x^9$ 4

(d) $1-4x^3-8x^6-32x^9$

(e) $1+4x^3-8x^6+32x^9$

(f) none of the above

$(1+8x^3)^{1/2}=1+\frac{1}{2}(8x^3)+\frac{(\frac{1}{2})(\frac{1}{2}-1)}{}(8x^3)^2$

$=1+4x^3-\frac{1/4}{2}(8x^3)^2$

$=1+4x^3-8x^6$

3) The coefficient of $x^6$ in the Maclaurin of $f(x)=\sqrt{4+x^2}$ is equal to

a) 1/512
b) 1/256
c) 1/128
d) 1/64
e) 1/32
f) none of the above

$f=(4+x^2)^{1/2}$

$=\left(4\left(1+\frac{x^2}{4}\right)\right)^{1/2}$

$=2\left(1+\frac{x^2}{4}\right)^{1/2}$

$=2\left(1+\frac{1}{2}\left(\frac{x^2}{4}\right)+\frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}\left(\frac{x^2}{4}\right)^2+\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}\left(\frac{x^2}{4}\right)^3+\cdots\right)$

Coeff of $x^6$ $\quad 2\left[\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}\right]\frac{x^6}{4^3}$

$=\frac{2(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{4^3\cdot 3\times 2\times 1}=\frac{1}{4^4\times 2}$

$=\frac{1}{256\times 2}=1/512$

4) $\sum_{n=2}^{\infty}\frac{n}{5^n}=$

a) 5/16
b) 9/80
c) 0
d) 3/5
e) 1/5
f) none of the above

$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$

diff

$\frac{1}{(1-x)^2}=\sum_{n=1}^{\infty}nx^{n-1}$

let $x=1/5$

$\frac{1}{(1-\frac{1}{5})^2}=\sum_{n=1}^{\infty}n\frac{1}{5^{n-1}}$

$\frac{1}{(\frac{4}{5})^2}=\sum_{n=1}^{\infty}\frac{n}{5^n\cdot 5^{-1}}$

$\frac{25}{16}=5\sum_{n=1}^{\infty}\frac{n}{5^n}$

$\frac{5}{16}=\sum_{n=1}^{\infty}\frac{n}{5^n}$

$\frac{5}{16}=\frac{1}{5}+\sum_{n=2}^{\infty}\frac{n}{5^n}$

$\frac{9}{80}=\frac{25-16}{80}\Leftarrow\frac{5}{16}-\frac{1}{5}=\sum_{n=2}^{\infty}\frac{n}{5^n}$

5) $\sum_{n=0}^{\infty} \frac{1}{2^n}\binom{1/2}{n} =$

(a) $\sqrt{3/2}$
(b) $-\sqrt{3/2}$
(c) $1/\sqrt{3}$
(d) $\sqrt{3}$
(e) 1
(f) none of the above

$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$

let $k = \frac{1}{2}$
$x = \frac{1}{2}$

then

$(1+\frac{1}{2})^{1/2} = \sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{2^n}$

$\sqrt{\frac{3}{2}} = (\frac{3}{2})^{1/2} =$

6) The sum of the first n terms of the series $L = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is $S_n$. Which one of the following is false

(a) $|L - S_{11}| \le 1/12$
(b ) the sign of $L - S_{253}$ is positive
(c) $\frac{1}{2} \le L \le 1$
(d) $\frac{5}{2} \le L \le \frac{7}{2}$
(e) the sign of $L - S_{202}$ is negative
(f) none of the above

By alt. series estimate thm

(a) $|L - S_{11}| \le \frac{1}{12}$ (first unused)

(b) Sign $(L - S_{253})$ = Sign (first unused)
$=$ Sign $(\frac{(-1)^{254}}{253}) = +$

(c) $S_1 = 1$ , $S_2 = 1 - \frac{1}{2} = \frac{1}{2}$ $\Rightarrow \frac{1}{2} < L \le 1$

(d)


7) The Taylor series of $f(x) = 1/x$ about $x = 2$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^{n+1}$
(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x-2)^{n+1}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{n+1}}(x-2)^{n+1}$
(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^{n+1}$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$
(f) none of the above

$f = \frac{1}{x} = x^{-1}$

$f' = (-1)x^{-2}$

$f'' = (-1)(-2)\,x^{-3}$

$f''' = (-1)(-2)(-3)\,x^{-4}$

$\vdots$

$f^{(n)}(x) = (-1)(-2)(-3)\cdots(-n)\,x^{-(n+1)}$

$f^{(n)}(2) = (-1)(-2)\cdots(-n)\,2^{-(n+1)}$
$= (-1)^n \cdot n!\, 2^{-(n+1)}$

Taylor $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot \cancel{n!}\, 2^{-(n+1)}}{\cancel{n!}} (x-2)^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

8) Using the binomial series, the coefficient of $x$ in the Maclaurin series of the function $\frac{1}{\sqrt[3]{1+2x}}$ is

(a) 0
(b) -1
(c) 1
(d) 2/3
(e) -2/3
(f) none of the above

$f = (1+2x)^{1/3}$ , $k = \frac{1}{3}$

$f = 1 + \frac{1}{3}(2x) + \ldots$

Coeff of $x$ is $\frac{2}{3}$