Name: KEY | ID:

MATH-102 Term-142 CQ-13 **FORM A**

(show your work and circle one letter to get a full mark or you will get zero)

1) The series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}3^{n-1}}{5^{n+1}}$ is

(a) -0.025
(b) -0.015
(c) -0.6
(d) 0.46
(e) 0.035
(f) none of the above

$$= \sum_{n=2}^{\infty} \frac{(-3)^{n-1}}{5^{n-1}\cdot 5^2}$$

$$= \sum_{n=2}^{\infty} \frac{1}{25}\left(\frac{-3}{5}\right)^{n-1}$$

geometric with $r = -3/5$

$$a = \frac{1}{25}\left(-\frac{3}{5}\right)$$

$$\text{Sum} = \frac{a}{1-r} = \frac{-\frac{3}{25\times5}}{1+\frac{3}{5}} = \frac{-\frac{3}{25}}{5+3}$$

$$= \frac{-3/25}{8} = -\frac{3}{25\times 8} = -\left(\frac{3}{100}\right)\frac{1}{2}$$

$$= (-0.03)\left(\frac{1}{2}\right) = (-0.03)(0.5)$$

$$\cancel{-0.0015} = -0.015$$

2) if $\{S_n\}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ then $S_n =$

(a) 0.25
(b) 0.15
(c) 0.7
(d) 0.49
(e) 0.499
(f) none of the above

partial fraction

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$\sum = \sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

telescoping

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)} = \frac{1}{2} - \lim_{n\to\infty}\frac{1}{n+2} = \frac{1}{2}$$

$$S_n = \sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots$$

$$\cdots + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2} - \frac{1}{n+2}$$

3) The series $\sum_{n=5}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n}\right)$ is

(a) -1/7
(b) 1/7
(c) 1/30
(d) -1/30
(e) 1
(f) none of the above

$$\sum_{n=5}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n+1} + \frac{1}{n+1} - \frac{1}{n}\right)$$

$$= \sum_{n=5}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n+1}\right) + \sum_{n=5}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n}\right)$$

two telescoping

First $= \frac{1}{7}$
Second $= \frac{1}{6}$
$\Rightarrow \text{Sum} = \frac{1}{7} + \frac{1}{6}$

$$= \frac{6+7}{42} = \frac{13}{42}$$

4) $+\frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} + \frac{1}{256} =$ Sum

(a) 39/54
(b) 255/256
(c) 59/(3x54)
(d) 255/(6x256)
(e) 129/(6x128)

$$= \frac{1}{4}\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64}\right]$$

$$= \frac{1}{4}\sum_{n=1}^{7}\left(-\frac{1}{2}\right)^{n-1}$$

$$a = 1, \quad r = -1/2$$

$$S_n = a\frac{1-r^n}{1-r} \Rightarrow S_7 = (1)\frac{1-(-\frac{1}{2})^7}{1-(-1/2)}$$

$$S_7 = \frac{1+\frac{1}{2^7}}{1+\frac{1}{2}} = \frac{2+\frac{1}{2^6}}{3}$$

$$= \frac{2^7+1}{3\times 2^6}$$

$$\text{Sum} = \frac{1}{4}\left(\frac{2^7+1}{3\times2^6}\right) = \frac{2^7+1}{3\times2^8} = \frac{129}{3\times256}$$

$$= \frac{129}{6\times128}$$