

(show all your work and circle one letter to get a full mark or you will get zero)

- 1) The interval of convergence of the power series
- $\sum_{n=1}^{\infty} \frac{(-2)^n}{n} (x-3)^n$
- is

10 points

$$R = \lim_{n \rightarrow \infty} \left| \frac{(-2)^{n+1} (x-3)^{n+1}}{n+1} \cdot \frac{n}{(-2)^n (x-3)^n} \right| = 2|x-3|$$

$$2|x-3| < 1 \Rightarrow |x-3| < 1/2 \rightarrow (5/2, 7/2)$$

$$x = 7/2 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ alt. Harmonic conv}$$

$$x = 5/2 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} \left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} \frac{(-1)^n}{2^n} = \sum_{n=1}^{\infty} \frac{1}{n} \text{ harmonic div}$$

(a) (5/4, 7/4]

(b) [5/4, 7/4]

(c) [5/2, 7/2]

(d) (5/2, 7/2)

(e) (5/2, 7/2]

(f) None of the above

$$\left[\frac{5}{2}, \frac{7}{2} \right]$$

(TRUE or FALSE)

20 points (4 each)

- 2) If the power series
- $f(x) = \sum_{n=0}^{\infty} c_n (x + \frac{3}{2})^n$
- has radius of convergence
- $R = 2$
- , then
- $(-\frac{7}{2}, \frac{1}{2}) = I$
-
- center =
- $-\frac{3}{2} \Rightarrow$
- interval
- $(-\frac{3}{2} - 2, -\frac{3}{2} + 2)$

Statement	True or False	Reason
(a) $\lim_{n \rightarrow \infty} \frac{c_n}{2^n} = 0$	(T) (F)	$x = -1 \in I \Rightarrow \sum_{n=0}^{\infty} \frac{c_n}{2^n} \text{ conv} \Rightarrow \lim_{n \rightarrow \infty} \frac{c_n}{2^n} = 0$
(b) $\lim_{n \rightarrow \infty} (-1)^n \left(\frac{5}{2}\right)^n c_n \neq 0$	(T) (F)	$x = -4 \notin I \rightarrow \sum_{n=0}^{\infty} c_n (-1)^n \left(\frac{5}{2}\right)^n \text{ div}$
(c) $\sum_{n=1}^{\infty} \frac{nc_n}{2^{n-1}}$ is convergent	(T) (F)	$f'(x) = \sum_{n=1}^{\infty} n c_n (x + \frac{3}{2})^{n-1}$ $x = -1 \in I \Rightarrow \sum_{n=1}^{\infty} \frac{nc_n}{2^{n-1}} \text{ conv}$
(d) $\sum_{n=1}^{\infty} n c_n$ is convergent	(T) (F)	let $x = -1/2 \in I$ in $f'(x)$ $\sum_{n=1}^{\infty} n c_n \text{ conv } -1/2 \in I$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} c_n}{n+1}$ is divergent	(T) (F)	$\int f(x) dx = C + \sum_{n=0}^{\infty} \frac{c_n (x + \frac{3}{2})^{n+1}}{n+1}$ $x = -1/2 \in I \rightarrow C + \sum_{n=0}^{\infty} \frac{c_n}{n+1} \text{ conv}$