Please note that these question are not encluded in the linear case. KEY

Name:	ID:	Sec: 10

MATH-102 Term-132 CQ11 (show all your work & write final answer in the box)

1) $\int_0^{1/2} \frac{x+2}{(x-1)(x^2+1)}\,dx$

$$\frac{x+2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

multiply by $(x-1)(x^2+1)$

$$A(x^2+1) + (Bx+C)(x-1) = x+2$$

$x=1$: $2A = 3 \to A = 3/2$

$x=0$: $A - C = 2 \to C = A-2 = \frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$

coeff of x^2: $A + B = 0 \to B = -A = -3/2$

$$\int \frac{x+2}{(x-1)(x^2+1)}\,dx = \int \frac{3/2}{x-1}\,dx + \int \frac{-\frac{3}{2}x - \frac{1}{2}}{x^2+1}\,dx$$

$$= \frac{3}{2}\int \frac{dx}{x-1} - \frac{3}{2}\int \frac{x}{x^2+1}\,dx - \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{2}\int \frac{dx}{x-1} - \frac{3}{4}\int \frac{2x}{x^2+1}\,dx - \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{2}\ln|x-1| - \frac{3}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

2) $\int_0^{1/2} \frac{1}{x^3+1}\,dx$

$$\frac{1}{(x^3+1)} = \frac{1}{(x+1)(x^2-x+1)}$$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

multiply by $(x+1)(x^2-x+1)$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$x=-1$: $1 = 3A \Rightarrow A = 1/3$

$x=0$: $1 = A + C \Rightarrow C = 1 - 1/3 = 2/3$

Coeff of x^2: $0 = A + B \Rightarrow B = -1/3$

$$\int \frac{1}{x^3+1}\,dx = \int \frac{1/3}{x+1}\,dx + \int \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{x}{x^2-x+1} + \frac{2}{3}\int \frac{dx}{x^2-x+1}$$

$$= \frac{1}{3}\ln|x+1| - \cdots$$

$$\int \frac{x}{x^2-x+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2-x+1}\,dx = \frac{1}{2}\int \frac{2x-1+1}{x^2-x+1}\,dx$$

$$= \frac{1}{2}\int \frac{2x-1}{x^2-x+1}\,dx + \frac{1}{2}\int \frac{dx}{x^2-x+1}$$

$$= \frac{1}{2}\ln|x^2-x+1| + \cdots$$

$$\int \frac{dx}{x^2-x+1} = \int \frac{dx}{(x-\frac{1}{2})^2 + \frac{3}{4}} \to \tan^{-1}$$

Ans:	Ans: