

Name:

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Sec: 10

MATH-102 Term-132

CQ10

(show all your work & write final answer in the box)

$$1) \int \frac{x^2}{(4-x^2)^{3/2}} dx = \frac{1}{3}$$

$$x = 2 \sin \theta \rightarrow dx = 2 \cos \theta d\theta$$

$$(4-x^2)^{3/2} = (2 \cos \theta)^3 = 8 \cos^3 \theta$$

$$I = \int \frac{4 \sin^2 \theta (2 \cos \theta d\theta)}{8 \cos^3 \theta}$$

$$= \int \frac{\sin^2 \theta}{\cos^2 \theta} d\theta = \int \tan^2 \theta d\theta$$

$$= \int (\sec^2 \theta - 1) d\theta$$

$$= \tan \theta - \theta + C = \frac{x}{\sqrt{4-x^2}} - \sin^{-1} \frac{x}{2} + C$$

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{x^2}{(4-x^2)^{3/2}} dx =$$

$$= \left[\frac{x}{\sqrt{4-x^2}} - \sin^{-1} \frac{x}{2} \right]_0^{\frac{1}{\sqrt{3}}}$$

$$= \left(\frac{1}{\sqrt{3}} - \sin^{-1} \frac{1}{2} \right) - (\sin^{-1} 0)$$

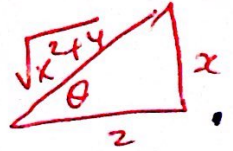
$$= \frac{1}{\sqrt{3}} - \frac{\pi}{6}$$

Ans: $\text{sech } x =$

$$2) \int \frac{1}{x\sqrt{x^2+4}} dx$$

$$x = 2 \tan \theta \rightarrow dx = 2 \sec^2 \theta d\theta$$

$$\sqrt{x^2+4} = 2 \sec \theta$$



$$I = \int \frac{2 \sec^2 \theta d\theta}{(2 \tan \theta)(2 \sec \theta)} = \frac{1}{2} \int \frac{\sec \theta d\theta}{\tan \theta}$$

$$= \frac{1}{2} \int \frac{1}{\sin \theta} d\theta = \frac{1}{2} \int \csc \theta d\theta$$

$$= \frac{1}{2} \ln |\csc \theta - \cot \theta| + C$$

$$= \frac{1}{2} \ln \left| \frac{\sqrt{x^2+4}}{x} - \frac{2}{x} \right| + C$$

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{x\sqrt{x^2+4}} dx = \frac{1}{2} \ln \left| \frac{\sqrt{1+4}}{1} - \frac{2}{1} \right| + C = \frac{1}{2} \ln \left| \frac{\sqrt{5}}{1} - 2 \right| + C$$

$$= \frac{1}{2} \ln \left| \frac{\sqrt{5}-2}{1} \right| + C$$

Ans: $y' =$