Q	MM	V1	V2	V3	V4
1	a	e	a	a	c
2	a	e	a	c	e
3	a	a	d	a	c
4	a	b	c	d	b
5	a	b	e	c	e
6	a	c	c	b	b
7	a	c	e	b	a
8	a	d	a	b	e
9	a	b	a	e	c
10	a	b	a	e	d
11	a	d	d	c	b
12	a	e	b	a	a
13	a	e	b	c	e
14	a	e	b	c	d
15	a	b	b	e	d
16	a	c	d	a	c
17	a	a	e	c	a
18	a	b	c	a	c
19	a	e	e	c	b
20	a	d	a	b	d

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 132
Sunday 02/03/2014
Net Time Allowed: 120 minutes

MASTER VERSION

- Detailed Solution
- Solution Key (last page) [Answer Key]

1. Suppose g is an integrable function. If $\int_2^{-2} g(t)\,dt = 3$ and $\int_5^{-2} g(t)\,dt = 4$, then $\int_2^5 g(t)\,dt =$

(a) -1

(b) 1

(c) 7

(d) -7

(e) 3

$-2 \qquad 2 \qquad 5$

$\int_{-2}^{5} g(t)\,dt = \int_{-2}^{2} g(t)\,dt + \int_{2}^{5} g(t)\,dt$

$-4 = -3 + \int_{2}^{5} g(t)\,dt$

$\Rightarrow \int_{2}^{5} g(t)\,dt = -4 + 3 = -1$

2. If $\int \frac{1}{\sqrt{x}\, e^{\sqrt{x}}} \sec^2(e^{-\sqrt{x}} + 1)\,dx = \int c \sec^2 u\,du$, then $cu =$

(a) $-2(e^{-\sqrt{x}} + 1)$

(b) $e^{-\sqrt{x}} + 1$

(c) $\frac{1}{\sqrt{x}\, e^{\sqrt{x}}}(e^{-\sqrt{x}} + 1)$

(d) $-\frac{1}{2}(e^{-\sqrt{x}} + 1)$

(e) $\frac{-2}{\sqrt{x}\, e^{\sqrt{x}}}(e^{-\sqrt{x}} + 1)$

$u = e^{-\sqrt{x}} + 1 \Rightarrow du = (e^{-\sqrt{x}} \cdot \frac{-1}{2\sqrt{x}})\,dx$

$\Rightarrow -2\,du = \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\,dx$

So $\int \frac{1}{\sqrt{x}\, e^{\sqrt{x}}} \sec^2(e^{-\sqrt{x}} + 1)\,dx$

$= \int -2 \sec^2 u\,du$

Thus $c = -2$ and so

$cu = -2(e^{-\sqrt{x}} + 1)$

3. $\int_0^3 8\sqrt{9-x^2}\,dx =$

$8\int_0^3 \sqrt{9-x^2}\,dx$

(a) 18π

(b) 8π

(c) 2π

(d) 36π

(e) 16π

$= 8\cdot\frac{1}{4}\pi(3)^2$

$= 18\pi$

4. $\int 14r^4(1-r^5)^6\,dr =$

Let $u = 1-r^5$. Then $du = -5r^4\,dr$

$\int 14r^4(1-r^5)^6\,dr = 14\int u^6\cdot\frac{-1}{5}\,du$

$= -\frac{14}{5}\int u^6\,du$

$= \frac{-14}{5}\cdot\frac{u^7}{7} + C$

$= -\frac{2}{5}u^7 + C$

$= -\frac{2}{5}(1-r^5)^7 + C$

(a) $-\frac{2}{5}(1-r^5)^7 + C$

(b) $\frac{1}{10}(1-r^5)^7 + C$

(c) $\frac{-2}{(1-r^5)^7} + C$

(d) $-\frac{1}{7}(1-r^5)^7 + C$

(e) $\frac{2}{7}(1-r^5)^6 + C$

5. Let b be a real number such that $0 < b < 2$. If the average value of $f(x) = \cos(2x)$ on the interval $[0, b]$ is **zero**, then $b =$

$f_{ave} = \frac{1}{b-0} \int_0^b \cos(2x)\, dx$

(a) $\frac{\pi}{2}$

$0 = \frac{1}{b} \cdot \frac{1}{2} \sin(2x) \Big]_0^b$

(b) $\frac{\pi}{3}$

$0 = \frac{1}{b} \cdot \left(\frac{1}{2} \sin(2b) - 0 \right)$

(c) $\frac{\pi}{4}$

$0 = \frac{1}{2b} \sin(2b)$

(d) $\frac{\pi}{5}$

$\Rightarrow \sin(2b) = 0$

$\Rightarrow 2b = n\pi$, n is integer

(e) $\frac{\pi}{6}$

$\Rightarrow b = \frac{n\pi}{2}$

Since $0 < b < 2$, we must take $n = 1$. So $b = \frac{\pi}{2}$

6. Let f be an **odd** and continuous function. If $\int_0^4 f(x)\, dx = 6$, then $\int_0^2 f(-2x)\, dx =$

Let $u = -2x$. Then $du = -2dx$.

(a) -3

$\int_0^2 f(-2x)\, dx = -\frac{1}{2} \int_0^{-4} f(u)\, du$

(b) -6

$= \frac{1}{2} \int_{-4}^0 f(u)\, du$

(c) 6

$= \frac{1}{2} \cdot (-6)$ Since f is odd

(d) -12

$= -3$

(e) 12

7. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \sin|2x|\, dx =$ $\int_{-\frac{\pi}{6}}^{0} \sin(-2x)\, dx + \int_0^{\pi/2} \sin(2x)\, dx$

$= -\int_{-\frac{\pi}{6}}^{0} \sin(2x)\, dx + \int_0^{\pi/2} \sin(2x)\, dx$

$= -\left(-\frac{1}{2}\cos(2x)\Big]_{-\frac{\pi}{6}}^{0}\right) + -\frac{1}{2}\cos(2x)\Big]_0^{\pi/2}$

$= -\left[-\frac{1}{2} + \frac{1}{4}\right] + \left(\frac{1}{2} + \frac{1}{2}\right)$

$= \frac{1}{4} + 1$

$= \frac{5}{4}$

(a) $\frac{5}{4}$

(b) $\frac{3}{4}$

(c) $1 - \frac{\sqrt{3}}{4}$

(d) $\frac{\sqrt{3}}{4}$

(e) $1 + \frac{1}{\sqrt{2}}$

8. The position function of a particle moving in a straight line is given by

$$s(t) = \frac{1}{2}t^2 - t, \quad t \geq 0$$

The **total distance** traveled by the particle over the time interval $0 \leq t \leq 2$ is

$v(t) = t - 1$

$D = \int_0^2 |v(t)|\, dt$

$= \int_0^2 |t-1|\, dt$

$= \int_0^1 (1-t)\, dt + \int_1^2 (t-1)\, dt$

$= t - \frac{1}{2}t^2\Big]_0^1 + \frac{1}{2}t^2 - t\Big]_1^2$

$= \left(1 - \frac{1}{2}\right) + \left(0 - \left(\frac{-1}{2}\right)\right)$

$= \frac{1}{2} + \frac{1}{2} = \underline{1}$

(a) 1

(b) $\frac{1}{3}$

(c) 3

(d) $\frac{5}{3}$

(e) 5

9. Let $y = f(x)$ be a curve that has a positive derivative and passes through the point $(1, 1)$. If its length L, from $x = 1$ to $x = 4$, is given by

$$L = \int_1^4 \sqrt{1 + \frac{16}{9\sqrt{x}}}\, dx,$$

then $f(2^{4/3}) =$

(a) $\frac{25}{9}$

(b) $\frac{5}{9}$

(c) $\frac{3}{4}$

(d) $\frac{16}{9}$

(e) $\frac{8}{3}$

$$\left(\frac{dy}{dx}\right)^2 = \frac{16}{9\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{3} \cdot \frac{1}{\sqrt[4]{x}}$$ (derivative is +)

$$\Rightarrow y = \frac{4}{3} \cdot \frac{4}{3} x^{3/4} + C$$

$$\Rightarrow 1 = \frac{16}{9} (1)^{3/4} + C \Rightarrow C = 1 - \frac{16}{9} = \frac{-7}{9}$$

So $f(x) = \frac{16}{9} x^{3/4} - \frac{7}{9}$

$$\Rightarrow f(2^{4/3}) = \frac{16}{9} \cdot 2 - \frac{7}{9} = \frac{32-7}{9} = \frac{25}{9}$$

10. The **slope** of the tangent line to the curve

$$y = \int_2^{\sec x} \frac{\sqrt{t^2 - 1}}{t}\, dt, \quad 0 < x < \frac{\pi}{2}$$

at $x = \frac{\pi}{3}$ is

(a) 3

(b) 2

(c) $\frac{1}{2}$

(d) $2\sqrt{3}$

(e) $\frac{\sqrt{3}}{4}$

$$y' = \frac{\sqrt{\sec^2 x - 1}}{\sec x} \cdot \frac{d}{dx}[\sec x]$$

$$= \frac{\tan x}{\sec x} \cdot \sec x \tan x$$

$$= \tan^2 x$$

$$\text{slope} = y'\Big|_{x=\frac{\pi}{3}} = \tan^2\left(\frac{\pi}{3}\right) = (\sqrt{3})^2 = 3$$

11. The **area** of the region enclosed by the curves $y = \sqrt{x}$ and $y = \frac{1}{8}x^2$ is

$\sqrt{x} = \frac{1}{8}x^2 \Rightarrow x = \frac{1}{64}x^4 \Rightarrow 64x = x^4$

$\Rightarrow x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0 \Rightarrow x = 0$ or $x = 4$

(a) $\frac{8}{3}$

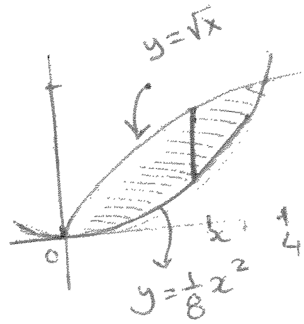
(b) $\frac{7}{4}$

(c) $\frac{9}{5}$

(d) $\frac{5}{2}$

(e) $\frac{11}{6}$

$$A = \int_0^4 \left(\sqrt{x} - \frac{1}{8}x^2\right) dx$$
$$= \frac{2}{3}x^{3/2} - \frac{1}{24}x^3 \Big]_0^4$$
$$= \frac{2}{3}\cdot 8 - \frac{1}{24}\cdot 64$$
$$= \frac{16}{3} - \frac{8}{3}$$
$$= \frac{8}{3}$$



Let $u = x-4$. Then $du = dx$

12. $\int \frac{x\ [redacted]}{(x-4)^3}\,dx = \int \frac{u+4}{u^3}\,du = \int u^{-2} + 4u^{-3}\,du$

$= \frac{u^{-1}}{-1} + 4\cdot\frac{u^{-2}}{-2} + C$

$= \frac{-1}{u} - \frac{2}{u^2} + C$

$= \frac{-1}{x-4} - \frac{2}{(x-4)^2} + C$

(a) $\frac{-1}{x-4} - \frac{2}{(x-4)^2} + C$

(b) $\ln|x-4| - \frac{4}{x-4} + C$

(c) $2\ln|x-4| - \frac{1}{(x-4)^2} + C$

(d) $\frac{1}{(x-4)^2} + \frac{1}{(x-4)^3} + C$

(e) $\frac{-1}{x-4} + \frac{2}{(x-4)^4} + C$

13. The curve $y = \sqrt{4x - x^2}$, $1 \le x \le 4$, is revolved about the x-axis. The **area of the generated surface** is equal to

$$y' = \frac{4-2x}{2\sqrt{4x-x^2}} = \frac{2-x}{\sqrt{4x-x^2}}$$

(a) 12π

(b) 6π

(c) 3π

(d) $4\pi - 2$

(e) 2π

$$S = \int_1^4 2\pi y \sqrt{1+(y')^2}\, dx$$

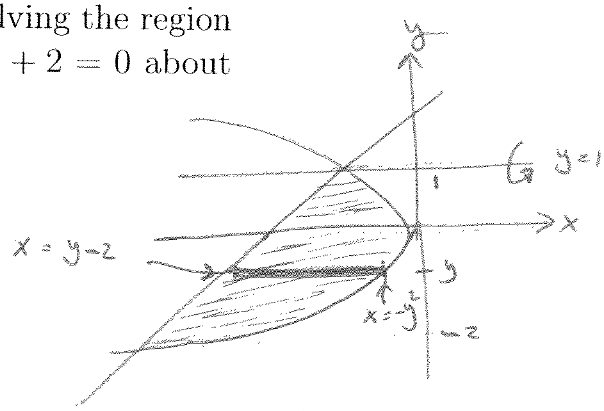
$$= \int_1^4 2\pi \cdot \sqrt{4x-x^2} \cdot \sqrt{1+\frac{(2-x)^2}{4x-x^2}}\, dx$$

$$= \int_1^4 2\pi \sqrt{(4x-x^2)+(2-x)^2}\, dx$$

$$= \int_1^4 2\pi \sqrt{4x-x^2+4-4x+x^2}\, dx$$

$$= \int_1^4 2\pi \cdot 2\, dx = 4\pi \int_1^4 dx = 12\pi$$

14. The **volume** of the solid generated by revolving the region bounded by the curves $y^2 = -x$ and $x - y + 2 = 0$ about the line $y = 1$ is given by the integral



(a) $\int_{-2}^{1} 2\pi(y^3 - 3y + 2)\, dy$

(b) $\int_{-4}^{0} 2\pi(y^3 + y^2 + 3y - 2)\, dy$

(c) $\int_{-2}^{1} 2\pi(-y^3 + 3y - 4)\, dy$

(d) $\int_{-4}^{0} 2\pi(-y^3 + y^2 - 3y + 2)\, dy$

(e) $\int_{-2}^{1} 2\pi(y^3 + 2y^2 - 3y - 2)\, dy$

By the Shell method,

$$V = \int_{-2}^{1} 2\pi \cdot (1-y) \cdot (-y^2-(y-2)) \cdot dy$$

$$= \int_{-2}^{1} 2\pi (1-y)(-y^2-y+2)\, dy$$

$$= \int_{-2}^{1} 2\pi (-y^2-y+2+y^3+y^2-2y)\, dy$$

$$= \int_{-2}^{1} 2\pi (y^3-3y+2)\, dy$$

15. The **length** of the curve

$$y = (1 - x^{2/3})^{3/2}, \quad \frac{1}{2} \le x \le 1$$

is

(a) $\frac{3}{2}\left(1 - \frac{1}{\sqrt[3]{4}}\right)$

(b) $\frac{2}{3} - \sqrt[3]{2}$

(c) 6

(d) $3\sqrt[3]{4} - 1$

(e) $6 - \frac{2}{\sqrt[3]{4}}$

• $y' = \frac{3}{2}(1 - x^{2/3})^{1/2} \cdot \frac{2}{3} x^{-1/3}$

• $(y')^2 = (1 - x^{2/3}) \cdot x^{-2/3}$

$= x^{-2/3} - 1$

• $1 + (y')^2 = x^{-2/3}$

• $\sqrt{1+(y')^2} = x^{-1/3}$

$$L = \int_{1/2}^{1} \sqrt{1+(y')^2}\, dx$$

$$= \int_{1/2}^{1} x^{-1/3}\, dx$$

$$= \frac{3}{2} x^{2/3} \Big]_{1/2}^{1}$$

$$= \frac{3}{2}\left(1 - \frac{1}{\sqrt[3]{4}}\right)$$

16. The **volume** of the solid generated by revolving the shaded region about the y-axis is

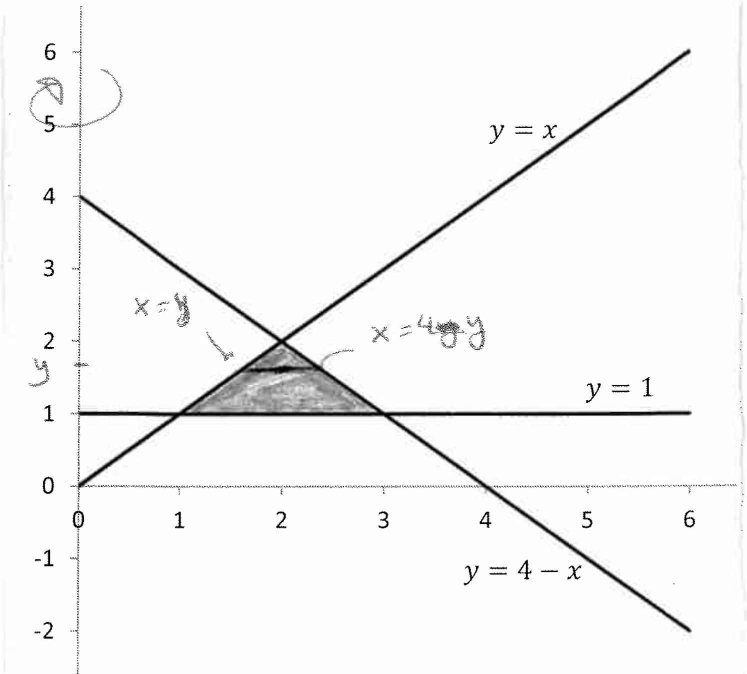
(a) $4\,\pi$

(b) $\frac{8\pi}{3}$

(c) $\frac{3\pi}{4}$

(d) $3\,\pi$

(e) $\frac{7\pi}{3}$



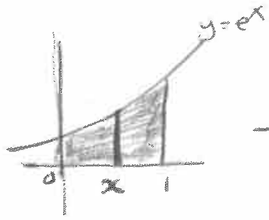
By the washer method:

$$V = \pi \int_1^2 (4-y)^2 - y^2\, dy$$

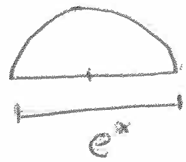
$$= \pi \int_1^2 16 - 8y\, dy = \pi \cdot 16y - 4y^2 \Big]_1^2 = \pi \cdot [(32-16) - (16-4)]$$

$$= \pi\,(16 - 12) = 4\pi$$

17. The base of a solid is bounded by the curves $y = e^x$, $y = 0$, $x = 0$, and $x = 1$. If the cross sections of the solid, perpendicular to the x-axis, are **semicircles**, then the **volume** of the solid is



Cross section $\longrightarrow$



radius $= \frac{1}{2} e^x$

(a) $\dfrac{\pi(e^2-1)}{16}$

(b) $\dfrac{\pi(e^2-1)}{8}$

(c) $\pi(e^4-1)$

(d) $\dfrac{\pi(e-1)}{2}$

(e) $\dfrac{\pi(e^2-2)}{8}$

$$V = \int_0^1 A(x)\,dx$$
$$= \int_0^1 \frac{1}{2}\pi\left(\frac{1}{2}e^x\right)^2 dx$$
$$= \frac{\pi}{8}\int_0^1 e^{2x}\,dx$$
$$= \frac{\pi}{16} e^{2x}\Big]_0^1 = \frac{\pi}{16}(e^2-1).$$

18. If $F(x) = \int_1^x \frac{\sin t}{t}\,dt$, $x > 0$, then $\int_1^5 \frac{\sin(2t)}{t}\,dt =$

(a) $F(10) - F(2)$

(b) $F(5)$

(c) $\frac{1}{2}F(5)$

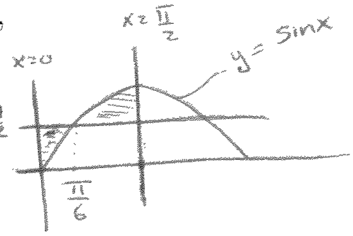
(d) $F(10) - F(5)$

(e) $2F(5) - F(2)$

Let $u = 2t$. Then $du = 2\,dt$.

$$\int_1^5 \frac{\sin(2t)}{t}\,dt = \int_2^{10} \frac{\sin u}{u}\,du$$
$$= F(u)\Big]_2^{10}$$
$$= F(10) - F(2)$$

19. The **area** of the region ~~enclosed~~ between by the curves $y = \sin x$, $y = \frac{1}{2}$, $\cancel{x = 0 \text{ and } x = \frac{\pi}{2}}$ is for $0 \le x \le \frac{\pi}{2}$ is



(a) $\sqrt{3} - 1 - \frac{\pi}{12}$

(b) $\sqrt{3} - \frac{\pi}{3}$

(c) $\sqrt{3} + 1 - \frac{\pi}{12}$

(d) $3\sqrt{3} - \frac{\pi}{2}$

(e) $\sqrt{3} - 2 + \frac{\pi}{6}$

$$A = \int_0^{\pi/6} \left(\tfrac{1}{2} - \sin x\right) dx + \int_{\pi/6}^{\pi/2} \left(\sin x - \tfrac{1}{2}\right) dx$$

$$= \tfrac{1}{2}x + \cos x \Big]_0^{\pi/6} + -\cos x - \tfrac{1}{2}x \Big]_{\frac{\pi}{6}}^{\pi/2}$$

$$= \left(\tfrac{\pi}{12} + \tfrac{\sqrt{3}}{2} - 1\right) + \left(-0 - \tfrac{\pi}{4}\right) - \left(-\tfrac{\sqrt{3}}{2} - \tfrac{\pi}{12}\right)$$

$$= \tfrac{\pi}{12} + \tfrac{\sqrt{3}}{2} - 1 - \tfrac{\pi}{4} + \tfrac{\sqrt{3}}{2} + \tfrac{\pi}{12}$$

$$= \sqrt{3} - 1 + \frac{1 - 3 + 1}{12}\pi$$

$$= \sqrt{3} - 1 - \frac{\pi}{12}$$

20. $\displaystyle\int \frac{x-3}{3+x^2}\,dx =$

$$\int \frac{x}{3+x^2} - \frac{3}{3+x^2}\,dx = \tfrac{1}{2}\ln(3+x^2) - 3\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$= \tfrac{1}{2}\ln(3+x^2) - \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C.$$

(a) $\frac{1}{2}\ln(x^2+3) - \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(b) $\frac{1}{2}\ln(x^2+3) - \frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(c) $\frac{1}{2}\ln(x^2+3) - \tan^{-1}\left(\frac{x}{3}\right) + C$

(d) $\ln(x^2+3) - \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(e) $\ln(x^2+3) - \frac{1}{\sqrt{3}}\tan^{-1}\left(x\sqrt{3}\right) + C$