King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 122
Wednesday 27/02/2013
Net Time Allowed: 120 minutes

MASTER VERSION

1. The average value of the function $f(x) = |2x| - 3$ over the interval $[-1, 2]$ is

(a) $\frac{-4}{3}$

(b) $\frac{1}{3}$

(c) 1

(d) -1

(e) $-\frac{1}{3}$

2. $\int_0^{\pi/8} \frac{\sin(2\theta)}{\cos^2(2\theta)}\, d\theta =$

(a) $\frac{1}{2}(\sqrt{2} - 1)$

(b) $2\sqrt{2} - 1$

(c) $\sqrt{2} + 1$

(d) $\frac{\sqrt{2}}{2} + 1$

(e) $\frac{\sqrt{2}}{2}$

3. $\int_0^{1/2} \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}\, dx =$

(a) $e^{\pi/6} - 1$

(b) $e^{\pi/4} + 1$

(c) $e^{\pi/4} - 1$

(d) $e^{\pi/3} + 1$

(e) $2e - 1$

4. The length of the curve $y = \frac{2}{3}(x-2)^{3/2}$ from $x = 2$ to $x = 5$ is

(a) $\frac{14}{3}$

(b) $\frac{16}{3}$

(c) $\frac{13}{3}$

(d) $\frac{17}{3}$

(e) $\frac{11}{3}$

5. $\int_0^1 (1+x)\sqrt{1-x}\,dx =$

(a) $\frac{14}{15}$

(b) $\frac{8}{15}$

(c) $\frac{3}{5}$

(d) $\frac{4}{3}$

(e) 1

6. Let $\int_1^{x^2} \frac{2f(\sqrt{t})}{t^2}\,dt = x^2 - 1$. If $x > 0$, then $f'(2) =$

(a) 16

(b) 18

(c) 20

(d) 22

(e) 24

7. $\int_{\pi/4}^{\pi/3} (\tan\theta + \cot\theta)^2 \, d\theta =$

(a) $\frac{2\sqrt{3}}{3}$

(b) $\sqrt{3}$

(c) $1 + \frac{\pi}{12}$

(d) $2 - \frac{\pi}{12}$

(e) $1 + \sqrt{3}$

8. Let f be an integrable function on $[-4, 7]$. If $\int_{-4}^{7} f(x)\,dx = 6$ and $\int_{1}^{7} (f(x) - 2)dx = -8$, then $\int_{-4}^{1} f(x)dx =$

(a) 2

(b) 1

(c) 4

(d) 3

(e) 5

9. The area of the region enclosed by the curve $y^2 = -x$ and the line $x + y + 2 = 0$ is equal to

(a) $\frac{9}{2}$

(b) 3

(c) $\frac{10}{2}$

(d) $\frac{3}{2}$

(e) 2

10. The volume of the solid generated by rotating the region enclosed by the curve $y = 2\sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the x-axis, is equal to

(a) 2π

(b) π

(c) $\frac{7\pi}{5}$

(d) $\frac{2\pi}{5}$

(e) $\frac{\pi}{5}$

11. The region enclosed by the curve $y = e^{x-1}$, and the lines $x = 1$, $x = 3$, and $y = 0$ is rotating about the x-axis,then the volume of the solid generated is equal to

(a) $\frac{\pi}{2}(e^4 - 1)$

(b) $\frac{\pi}{2}(e^4 + 2)$

(c) $\frac{\pi}{2}(e^2 - 2)$

(d) $\frac{\pi}{2}(e^4 - 3)$

(e) $\pi(e^6 - e^2)$

12. $\int_0^{e-1} \frac{1}{(1+t)(1+2\ln\sqrt{t+1})}\,dt =$

(a) $\ln 2$

(b) $e \ln 2$

(c) $\frac{1}{e}\ln 2$

(d) $e - \ln 2$

(e) $\ln 2 - \frac{1}{e}$

13. If $F'(x) = \sin x - \cos x$, then the net change in the function $F(x)$ over the interval $0 \leq x \leq \dfrac{\pi}{2}$ is equal to

(a) 0

(b) 1

(c) $4\sqrt{2} - 1$

(d) $\dfrac{\sqrt{2}}{2}$

(e) $\dfrac{\sqrt{2}}{4}$

14. $\displaystyle\int_{3\pi/4}^{\pi/4} \frac{\cos(2x)}{1 + \sin^2(2x)}\, dx =$

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{3\pi}{4}$

(c) $\dfrac{\pi}{2}$

(d) π

(e) $\dfrac{3\pi}{2}$

15. The area of the region enclosed by the curve $y = \pi \sin(2\pi x)$ and the x-axis between $x = 0$ and $x = \dfrac{3}{4}$ is equal to:

(a) $\dfrac{3}{2}$

(b) 2

(c) $\dfrac{5}{2}$

(d) $\dfrac{1}{2}$

(e) 3

16. The base of a solid is a triangular region bounded by the lines $y = x$, $y = 1$, and $x = 0$. If the cross-sections of the solid perpendicular to the y-axis are semi-circles with diameters running across the base of the solid, then the volume of the solid is equal to

(a) $\dfrac{\pi}{24}$

(b) $\dfrac{3\pi}{8}$

(c) $\dfrac{\pi}{16}$

(d) $\dfrac{\pi}{36}$

(e) $\dfrac{\pi}{4}$

17. Let P be a partition of the interval $[0, 2]$, then the limit $\lim_{\|p\|\to 0} \sum_{k=1}^{n} \left[c_k + \sqrt{4 - c_k^2}\right] \triangle x_k =$

[Hint: You may use known areas]

(a) $\pi + 2$

(b) $\pi - 1$

(c) $2\,\pi$

(d) $4\pi + 2$

(e) $\pi + 1$

18. The length of the curve $x = \dfrac{y^2}{2} - \dfrac{\ln y}{4}$ from $y = 1$ to $y = e$ is

(a) $\dfrac{1}{2}e^2 - \dfrac{1}{4}$

(b) $\dfrac{3}{4}e^2$

(c) $\dfrac{1}{2}e^2 - 2$

(d) e^2

(e) $\dfrac{1}{2}e^2 - 1$

19. The area in the first quadrant enclosed by the lines $y = 2x$, $y = \frac{1}{2}x$, and $y = -x + 6$ is equal to

(a) 6

(b) 8

(c) 10

(d) 12

(e) 14

20. The region in the first quadrant enclosed by the parabolas $y = x^2$, $y = 2 - x^2$, and the y-axis is rotating about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^1 4\pi(1 + x - x^2 - x^3)\,dx$

(b) $\int_0^2 2\pi(1 + x - x^2 - x^3)\,dx$

(c) $\int_0^1 4\pi(1 - 2x - 2x^2 + x^3)\,dx$

(d) $\int_0^2 2\pi(1 - x - x^2 - x^3)\,dx$

(e) $\int_0^1 2\pi(4 - x^2 + 2x^4)\,dx$