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Sec: 11(12:50-1:40) 14(1:50-2:40)

MATH-102

Term-141

Class-QUIZ-on-Power Series and Mac

(show all your work and circle one letter to get a full mark or you will get zero)

- 1) The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 2^n}$ is

- a) 1
b) 2
c) 3
d) 4
e) 5

- f) none of the above

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-1)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x-1)^n} \right|$$

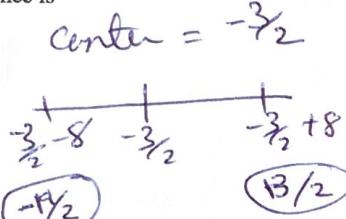
$$= |x-1| \cdot \frac{n}{2(n+1)}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{2} |x-1| < 1$$

$$\Rightarrow |x-1| < 2 \Rightarrow R=2$$

- 2) if the radius of the convergence of $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{n4^{2n}}$ is 8, then the interval of convergence is

- a) $(-19/2, 13/2]$
b) $(-19/2, 13/2)$
c) $[-19/2, 13/2]$
d) $[-19/2, 13/2)$
e) $[-13/2, -19/2)$
f) none of the above



$$x = \frac{13}{2} \Rightarrow \sum \frac{(16)^n}{n4^{2n}} = \sum \frac{1}{n} \text{ div}$$

$$x = -\frac{19}{2} \Rightarrow \sum \frac{(-16)^n}{n4^{2n}} = \sum \frac{(-1)^n}{n} \text{ converg}$$

- 3) The coefficient of x^{12} in the Maclaurin of $f(x) = 6x^2 \sin(x^2)$ is equal to

- a) 1
b) 0
c) -1
d) $1/20$
e) $1/10$
f) none of the above

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

$$6x^2 \sin x^2 = \sum_{n=0}^{\infty} \frac{6(-1)^n x^{4n+4}}{(2n+1)!}$$

coeffe of x^{12} when $n=2 \Rightarrow \frac{6(-1)^2 x^{12}}{5!} = \frac{6}{5 \times 4 \times 3 \times 2} x^{12} = \frac{1}{20} x^{12}$

- 4) The coefficient of x^5 in the Maclaurin of $f(x) = \frac{3x^3}{(x-3)^2}$ is equal to

- a) $1/9$
b) $-1/9$
c) $5/3^7$
d) $5/3^5$
e) 3^{-5}

- f) none of the above

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{3-x} = \frac{1}{3} \frac{1}{1-\frac{x}{3}}$$

$$\frac{1}{3-x} = \frac{1}{3} \sum_{n=0}^{\infty} \frac{x^n}{3^n} = \sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$$

$$\frac{d}{dx} \left[\frac{1}{3-x} \right] = (-1)(3-x)^{-2} = \frac{1}{(3-x)^2}$$

$$\frac{1}{(x-3)^2} = \sum_{n=0}^{\infty} \frac{nx^{n-1}}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{nx^{n-1}}{3^{n+1}}$$

multiplies by $3x^2$

$$\frac{3x^3}{(x-3)^2} = \sum_{n=1}^{\infty} \frac{nx^{n+2}}{3^n}$$

coeff of x^5 when $n=3$

$$n=3 \Rightarrow \frac{3x^5}{3^3} = \frac{3}{3^3} x^5 = \frac{1}{3^2} x^5 = \frac{1}{9} x^5$$