

Name: KEY

ID:

Sec: 11<sub>(12:50-1:40)</sub> 14<sub>(1:50-2:40)</sub>

MATH-102

Term-141

Class-QUIZ-on-Series

(show all your work and circle one letter to get a full mark or you will get zero)

1) The series  $\sum_{n=1}^{\infty} \frac{(-n!)^4}{(4n+3)!}$  is

- a) conditionally convergent  
 b) a divergent p series  
 c) divergent by the ratio test  
 d) a series for which the ratio test is inconclusive  
 e) absolutely convergent  
 f) none of the above

$$|a_n| = \frac{(n!)^4}{(4n+3)!} = \frac{(n!)^4}{(4n+7)(4n+6)(4n+5)(4n+4)}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{((n+1)!)^4}{(4n+7)!} \cdot \frac{(4n+3)!}{(n!)^4}$$

2) The series  $\sum_{n=1}^{\infty} \frac{3^n n^n}{2^{2n+1}}$  is

$$(a_n)^{\frac{1}{n}} = \frac{3n}{2^{2+n}}$$

- a) diverges by the root test  
 b) a convergent p series  
 c) converges by the root test  
 d) a series for which the root test is inconclusive  
 e) a divergent geometric series  
 f) none of the above

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \infty$$

3) The series  $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+3} - \sqrt{n+2})$  is

- a) diverges by the limit comparison test  
 b) conditionally convergent  
 c) absolutely convergent  
 d) diverges by the divergent test  
 e) divergent by the ratio test  
 f) none of the above

$$= \sum \frac{1}{\sqrt{n+3} + \sqrt{n+2}}$$

$$\text{to } \sum \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\frac{1}{\sqrt{n+3} + \sqrt{n+2}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+3} + \sqrt{n+2}}{\sqrt{n}} = 2$$

4) The series  $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{n+1} + 2)\sqrt{n}}$  is

- a) diverges by the limit comparison test  
 b) convergent by the integral test  
 c) convergent by the ratio test  
 d) convergent by the root test  
 e) divergent by the ratio test  
 f) none of the above

$$\lim_{n \rightarrow \infty} \frac{y_n}{\frac{1}{(\sqrt{n+1} + 2)\sqrt{n}}} = \lim_{n \rightarrow \infty} (\sqrt{n+1} + 2)\sqrt{n} = 1 \text{ bot divg}$$

5) The series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^{n+1} + (n+2)^3}$  is

$$|a_n| = \frac{1}{4^{n+1} + (n+2)^3}$$

- a) conditionally convergent  
 b) divergent  
 c) absolutely convergent  
 d) convergent by the integral test  
 e) divergent by the alternating series test  
 f) none of the above

$$\frac{1}{(n+2)^3 + 4^{n+1}} < \frac{1}{4^{n+1}}$$

by comparison test  $\sum k_n$  converges

3) The series  $\sum_{n=1}^{\infty} (3\sqrt{3} - \sqrt[n]{3})^{\frac{n}{2}}$  is

- a) the root test is inconclusive  
 b) conditionally convergent  
 c) a divergent geometric series  
 d) convergent by the root test  
 e) divergent by the root test  
 f) none of the above

$$(a_n)^{\frac{1}{n}} = (3\sqrt{3} - \sqrt[n]{3})^{\frac{1}{2}}$$

$$\lim (a_n)^{\frac{1}{n}} = (3\sqrt{3} - 0)^{\frac{1}{2}} = \sqrt{3\sqrt{3}} > 1$$

diverges by root test

# KEY

7) If the sum of the first n terms of a series  $\sum_{n=1}^{\infty} a_n$  is given by

$$S_n = \frac{2n}{n+1} \quad \text{then} \quad a_{10} =$$

- a) 1/110
- b) 4/110
- c) 2/120
- d) 2/101
- e) 2/101
- f) none of the above

$$a_n = S_n - S_{n-1}$$

$$\begin{aligned} a_{10} &= S_{10} - S_9 \\ &= \frac{20}{11} - \frac{18}{10} \\ &= \frac{200 - 198}{110} = \frac{2}{110} \end{aligned}$$

8) The series  $\sum_{n=2}^{\infty} \frac{6}{n^2 - 1} = 3 \sum_{n=1}^{\infty} \frac{2}{n^2 - 1} = 3 \sum_{n=1}^{\infty} \left( \frac{1}{n-1} - \frac{1}{n+1} \right)$

- a) 9/2
- b) 3
- c) 3/2
- d) 0
- e) 6
- f) none of the above

$$\begin{aligned} &= 3 \left\{ \sum_{n=2}^{\infty} \left( \frac{1}{n-1} - \frac{1}{n} \right) + \sum_{n=2}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) \right\} \\ &= 3 \left[ 1 - 0 + \frac{1}{2} - 0 \right] \\ &= 3 \left[ \frac{3}{2} \right] = \frac{9}{2} \end{aligned}$$

10) The minimum number of terms needed to estimate the sum

of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+3)^4}$  within 0.0001 (error) is

- a) n=5
- b) n=7
- c) n=10
- d) n=13
- e) n=6
- f) none of the above

$$|U_n| = \frac{1}{(2n+3)^4}$$

$$|U_{n+1}| \leq \frac{1}{10000}$$

$$\frac{1}{(2n+5)^4} \leq \frac{1}{10000}$$

$$(2n+5)^4 \geq 10000$$

$$2n+5 \geq 10$$

$$2n \geq 5 \Rightarrow n \geq 2.5$$

$$n \geq 3$$

The smallest choice is n=5

9) If  $\{S_n\}$  is the sequence of partial sums of the series

$$\sum_{n=3}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) \text{ then } S_n =$$

- a)  $\frac{1}{3} - \frac{1}{n+1} + \frac{1}{4} - \frac{1}{n+2}$
- b)  $2 - \frac{1}{n+1} - \frac{1}{n+2}$
- c)  $\frac{1}{3} - \frac{1}{n} + \frac{1}{4} - \frac{1}{n+2}$
- d)  $\frac{1}{6} - \frac{1}{n+1} - \frac{1}{n+2}$
- e)  $\frac{1}{3} - \frac{1}{n} + \frac{1}{4} - \frac{1}{n+1}$
- f) none of the above

$$\sum_{n=3}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) = \sum_{n=3}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) + \sum_{n=3}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$\text{Here } S_n = \left( \frac{1}{3} - \frac{1}{n+1} \right) + \left( \frac{1}{4} - \frac{1}{n+2} \right)$$