

Name:

KEY

MATH-102

ID:

Term-141

KEY

Sec: 11(12:50-1:40) 14(1:50-2:40)

Class-QUIZ-on-Taylor binomial mac

(show all your work and circle one letter to get a full mark or you will get zero)

1) The first three nonzero terms of the Taylor series

$$\text{of } f(x) = \sqrt{x}$$

about $a = 1$ are given by

- a) $1 - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3$
 b) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$
 c) $1 + \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3$
 d) $1 - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + \frac{1}{16}(x-1)^3$
 e) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$
 f) none of the above

$$f = x^{\frac{1}{2}}$$

$$f' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'' = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$f''' = \frac{3}{8}x^{-\frac{5}{2}}$$

4) If the Maclaurin series of $e^x \cos x$ is

$$A + Bx + Cx^2 + Dx^3 + \dots \text{ then } C+D =$$

a) $-1/6$

b) $1/6$

c) $-1/2$

d) $-1/3$ (circled)

e) $1/3$

f) none of the above

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots$$

$$C = \text{coeff of } x^2 = -\frac{1}{2} + \frac{1}{2} = 0$$

$$D = \text{coeff of } x^3 = 0 + \frac{1}{2} + \frac{1}{6} = \frac{-3+1}{6} = -\frac{2}{6} = -\frac{1}{3}$$

$$f(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$$

$$\begin{aligned} f(1) &= 1 \rightarrow 1 \\ f'(1) &= \frac{1}{2} \rightarrow y_2 \\ f''(1) &= -\frac{1}{4} \rightarrow -y_4 \\ f'''(1) &= \frac{3}{8} \rightarrow y_{16} \end{aligned}$$

2) if the radius of the convergence of $\sum_{n=1}^{\infty} \frac{(2x-3)^n}{n4^{2n}}$ is 8, then

the interval of convergence is

- a) $(-13/2, 19/2]$
 b) $(-13/2, 19/2)$
 c) $[-13/2, 19/2]$
 d) $(-13/2, 19/2)$ (circled)
 e) $[-13/2, -19/2)$
 f) none of the above

$$\text{center} = \frac{3}{2}$$

$$R = 8$$

$$\begin{array}{ccccccc} \frac{3}{2} & + & 1 & + & \frac{3}{2} & + & 8 \\ -8 & & \frac{11}{2} & & 11 & & 19/2 \end{array}$$

$$x = \frac{19}{2} \Rightarrow \sum \frac{1}{n} \text{ diverges}$$

$$x = -\frac{13}{2} \Rightarrow \sum \frac{(-1)^n}{n} \text{ converges}$$

3) The coefficient of x^6 in the Maclaurin of $f(x) = \sqrt{4+x^2}$ is equal to

$$\begin{aligned} f(x) &= (4+x^2)^{\frac{1}{2}} = \left(4 \left[1 + \frac{x^2}{4}\right]\right)^{\frac{1}{2}} \\ &= 2 \left(1 + \frac{x^2}{4}\right)^{\frac{1}{2}} \quad [k = \frac{1}{2}] \\ &= 2 \left(1 + k \left(\frac{x^2}{4}\right) + \frac{k(k-1)}{2!} \left(\frac{x^2}{4}\right)^2 + \dots\right) \end{aligned}$$

$$\text{Coeff of } x^6 \text{ is } 2 \left(\frac{k(k-1)(k-2)}{3!} \right) \left(\frac{x^2}{4}\right)^3$$

$$\frac{2 \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right)}{3!} \frac{x^6}{4^3} = +\frac{1}{4}$$

5) Using the binomial series we have for $|x| < 1/2$

$$f(x) = x\sqrt{4+32x^3}$$

$$\sqrt{4+32x^3} = 2\sqrt{1+8x^3}$$

$$a) 2 - 8x^3 + 16x^6 + 64x^9 + \dots$$

$$b) 2 + 8x^3 + 16x^6 + 64x^9 + \dots$$

$$c) 2 + 8x^3 - 16x^6 + 64x^9 + \dots$$

$$d) 2 + 8x^3 - 16x^6 - 64x^9 + \dots$$

$$e) 2 + 8x^3 + 16x^6 + 64x^9 + \dots$$

f) none of the above

$$\begin{aligned} (1+8x^3)^{\frac{1}{2}} &= 1 + \frac{1}{2}(8x^3) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(8x^3)^2 \\ &\quad + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(8x^3)^3 + \dots \end{aligned}$$

6) Express the integral of the function $f(x) = e^{x^2}$ as a power series then find the coefficient of x^4

a) -1

b) 0 (circled)

c) $1/24$

d) $1/10$

e) $1/42$

f) none of the above

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

$$e^{x^2} = 1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + \dots$$

$$\int e^{x^2} = x + \frac{1}{3}x^3 + \frac{1}{10}x^5 + \frac{1}{42}x^7 + \dots$$

7) $\sum_{n=2}^{\infty} \frac{1}{n!}$

- a) e
- b) 0
- c) 1
- d) e-2**
- e) 2+e
- f) none of the above

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\text{let } x=1$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$$e = 1 + 1 + \sum_{n=2}^{\infty} \frac{1}{n!}$$

$$e = 2 + \sum_{n=2}^{\infty} \frac{1}{n!}$$

$$e-2 = \sum_{n=2}^{\infty} \frac{1}{n!}$$

9) $\sum_{n=2}^{\infty} \frac{n}{5^n}$

- a) 5/16**
- b) 9/80
- c) 0
- d) 3/5
- e) 1/5
- f) none of the above

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

diff

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$$

$$\text{let } x=5$$

$$\frac{1}{(1-\frac{1}{5})^2} = \sum_{n=1}^{\infty} \frac{n}{5^{n-1}}$$

10) The coefficient of x^6 in the Maclaurin of $f(x) = \sqrt{4+x^2}$ is equal to

- a) 1/512
- b) 1/256
- c) 1/128
- d) 1/64
- e) 1/32
- f) none of the above

8) $\int_0^1 \frac{dx}{1-x^9} =$

a) $-1 + \sum_{n=0}^{\infty} \frac{1}{9n+1}$

b) $\sum_{n=0}^{\infty} \frac{1}{9n}$

c) $\sum_{n=0}^{\infty} \frac{1}{9n-1}$

d) $\sum_{n=0}^{\infty} \frac{1}{9n+1}$

e) $\sum_{n=1}^{\infty} \frac{1}{9n+1}$

- f) none of the above

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1-x^9} = \sum_{n=0}^{\infty} x^{9n}$$

$$\int \frac{dx}{1-x^9} = C + \sum_{n=0}^{\infty} \frac{x^{9n+1}}{9n+1}$$

$$\int \frac{dx}{1-x^9} = \sum_{n=0}^{\infty} \frac{1}{9n+1} - C$$

$$(1-x)^{-\frac{1}{9}} = (1-x)^{-1} + \dots$$

div'd by 5

$$\frac{1}{5(1-\frac{1}{5})^2} = \sum_{n=1}^{\infty} \frac{n}{5^n}$$

$$\frac{1}{5(1-\frac{1}{5})^2} = \frac{1}{5} + \sum_{n=2}^{\infty} \frac{n}{5^n}$$

$$\frac{1}{5(1-\frac{1}{5})^2} - \frac{1}{5} = \sum_{n=2}^{\infty} \frac{n}{5^n}$$

$$\frac{9}{80} =$$