1. $\int \frac{x^3 - 1}{x^3 - x^2}\, dx =$

(a) $x - \frac{1}{x} + \ln|x| + C$

(b) $\ln|x^3 - x^2| + C$

(c) $\frac{1}{2}x^2 - \ln|x| + C$

(d) $x + \frac{2}{x} - \ln|x| + C$

(e) $\frac{1}{x} - \frac{1}{x^2} + C$

2. $\int_0^{\frac{\pi}{4}} (\sec x + \tan x)^2\, dx =$

(a) $2\sqrt{2} - \frac{\pi}{4}$

(b) $\pi - \sqrt{2}$

(c) $\frac{\pi}{4} - 2$

(d) $2 + \sqrt{2} - \pi$

(e) $2\sqrt{2} + \pi$

3. Let $g(x) = \int_{kx}^{x} e^{t^2}\, dt$, where k is a constant. If $g'(1) = e$, then $k =$

(a) 0

(b) 1

(c) -1

(d) 2

(e) $\frac{1}{2}$

4. $\int_0^{\frac{\pi}{2}} \frac{e^x(\sin x - \cos x)}{1 + 3e^{2x}\cos^2 x}\, dx =$

[**Hint:** Let $u = \sqrt{3}\, e^x \cos x$]

(a) $\frac{\pi\sqrt{3}}{9}$

(b) $\frac{-\pi\sqrt{3}}{3}$

(c) $\frac{\pi\sqrt{3}}{6}$

(d) $-\pi\sqrt{3}$

(e) $\frac{2\pi\sqrt{3}}{9}$

5. If $\int \sqrt[3]{\sec x}\,\tan^3 x\,dx = A\sec^{\alpha} x + B\sec^{\beta} x + C,$ where α, β, A, B are real numbers, and C is an arbitrary constant, then the product $\alpha\beta A\,B$ is equal to

(a) -1

(b) 0

(c) -2

(d) $\frac{1}{2}$

(e) $-\frac{1}{7}$

6. $\int_0^{\frac{\pi^2}{4}} \cos(\sqrt{x})\,dx =$

(a) $\pi - 2$

(b) $2\,\pi - 1$

(c) $\frac{\pi}{2} + 1$

(d) $-\frac{\pi}{2} - 2$

(e) $\pi - \frac{3}{2}$

7. $\int \sqrt{36-x^2}\,dx =$

(a) $18\sin^{-1}\left(\frac{x}{6}\right)+\frac{1}{2}x\sqrt{36-x^2}+C$

(b) $18x\sin^{-1}\left(\frac{x}{6}\right)+C$

(c) $9\sqrt{36-x^2}\,\sin^{-1}\left(\frac{x}{6}\right)+C$

(d) $9\sin^{-1}\left(\frac{x}{6}\right)-2x\sqrt{36-x^2}+C$

(e) $18\sin^{-1}\left(\frac{x}{6}\right)+\sqrt{36-x^2}+C$

8. The improper integral $\int_e^{\infty}\frac{1}{x(1+\ln x)^2}\,dx$ is

(a) convergent and its value is $\frac{1}{2}$

(b) convergent and its value is 1

(c) convergent and its value is 0

(d) convergent and its value is $\frac{1}{4}$

(e) divergent

9. $\displaystyle\int \frac{2x-5}{x^2+x-2}\,dx =$

(a) $3\ln|x+2| - \ln|x-1| + C$

(b) $3\ln\left|\dfrac{x+2}{x-1}\right| + C$

(c) $\dfrac{1}{3}\ln\left|\dfrac{x-1}{x+2}\right| + C$

(d) $\ln|x^2+x-2| + C$

(e) $3\ln|x+2| - 4\ln|x-1| + C$

10. $\displaystyle\int \frac{\cos x}{1-\cos x}\,dx =$

(a) $-\csc x - \cot x - x + C$

(b) $-\ln|1-\cos x| + C$

(c) $\dfrac{\sin x}{x-\sin x} + C$

(d) $\sec x + \tan x - x + C$

(e) $\cos x - \sec x + C$

11. The **area** of the region bounded by the curves $y = \ln(x-1)$, $y = \ln(1-x), y = \ln 2$, $y = 0$ is

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) e^2

(e) $2\ln 2$

12. The **volume** of the solid generated by revolving the region in the first quadrant bounded by the graphs of $y = \sqrt{\cosh\left(\frac{x}{2}\right)}$, $x = \ln 4, x = 0$, $y = 0$ about the x-axis is

(a) $\frac{3\pi}{2}$

(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{4\pi}{3}$

(e) $\frac{\pi}{6}$

13. The **length** of the curve

$$y = \left(x^2 + \frac{2}{3}\right)^{3/2}, \quad 1 \le x \le 2$$

is

(a) 8

(b) 10

(c) 6

(d) 12

(e) 14

14. The **volume** of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = x + 2$ about the line $x = 3$ is given by

(a) $\int_{-1}^{2} 2\pi(3 - x)(x + 2 - x^2)\, dx$

(b) $\int_{-1}^{2} 2\pi(x - 3)(x + 2 - x^2)\, dx$

(c) $\int_{-1}^{2} 2\pi\, x(x + 2 - x^2)\, dx$

(d) $\int_{-1}^{2} 2\pi(3 - x)(x^2 - x + 2)\, dx$

(e) $\int_{-1}^{2} 2\pi(x + 3)(x^2 - x + 2)\, dx$

15. Which one of the following statements is **TRUE:**

(a) $\{a_n\}_{n=1}^{\infty}$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges

(b) $\{a_n\}_{n=1}^{\infty}$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges

(c) $\sum_{n=1}^{\infty} a_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{a_n}$ converges $(a_n \neq 0 \, \text{for} \, n \geq 1)$

(d) $\sum_{n=1}^{\infty} a_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n^2$ converges

(e) $\sum_{n=1}^{\infty} |a_n|$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges conditionally

16. The sequence $\frac{2 \cdot 1}{4 \cdot 4}, \frac{4 \cdot 3}{8 \cdot 7}, \frac{8 \cdot 5}{16 \cdot 10}, \frac{16 \cdot 7}{32 \cdot 13}, \ldots$

(a) converges to $\frac{1}{3}$

(b) converges to $\frac{1}{4}$

(c) converges to $\frac{3}{2}$

(d) converges to $\frac{5}{3}$

(e) diverges

17. The interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{n(x+2)^n}{5^{n+1}}$ is

(a) $(-7, 3)$

(b) $[-7, 3)$

(c) $(-7, 3]$

(d) $[-7, 3]$

(e) $(-3, -1)$

18. If $\mathcal{S}$ is the sum of the series

$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{5}{6}\right)^{2n+1} \frac{\pi^{2n+1}}{(2n+1)!}$$

then $2\mathcal{S} - 1 =$

(a) 0

(b) $-\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 1

(e) $-\frac{1}{4}$

19. The series $\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$ is

(a) divergent

(b) convergent by the comparison test

(c) convergent by the limit comparison test

(d) convergent by the integral test

(e) conditionally convergent

20. The sum of the series $\sum_{n=2}^{\infty} \frac{\ln(n+1) - \ln n}{\ln\, n \cdot \ln(n+1)}$ is

(a) $\frac{1}{\ln 2}$

(b) 0

(c) 1

(d) $\frac{-1}{\ln 2}$

(e) $\ln 3 - \ln 2$

21. The coefficient of x^4 in the Maclaurin series of $\dfrac{x}{\sqrt[3]{1+x}}$ is

$\left[\textbf{Hint:} \text{ you may use the binoimial series of} \dfrac{1}{\sqrt[3]{1+x}}\right]$

(a) $-\dfrac{14}{81}$

(b) $\dfrac{2}{9}$

(c) $-\dfrac{16}{81}$

(d) $-\dfrac{19}{54}$

(e) $\dfrac{17}{84}$

22. If $\sum\limits_{n=0}^{\infty} a_n(x-3)^n$ is the Taylor series for $f(x) = x^3 - 10x^2 + 6$ about $x = 3$, then $\sum\limits_{n=0}^{100} a_n =$

(a) -90

(b) -100

(c) -52

(d) -80

(e) -102

23. The series $\sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{1+\frac{n}{2}}}{e^{3n}}$ is

(a) convergent and its sum is $\frac{2e^3}{e^3+\sqrt{2}}$

(b) convergent and its sum is $1+\sqrt{2}\,e^{-3}$

(c) convergent and its sum is $\frac{e^3}{\sqrt{2}}$

(d) convergent and its sum is $1-\frac{e^3}{\sqrt{2}}$

(e) divergent

24. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ is

(a) conditionally convergent

(b) absolutely convergent

(c) a convergent $p-$ series

(d) divergent by the ratio test

(e) divergent by the $n^{th}-$ term test for divergence

25. For some suitable values of x, the Maclaurin series for $f(x) = \dfrac{27}{3+2x}$ is given by

(a) $\sum_{n=0}^{\infty} (-1)^n \dfrac{2^n\, x^n}{3^{n-2}}$

(b) $\sum_{n=0}^{\infty} \dfrac{2^n \cdot x^n}{3^n}$

(c) $\sum_{n=0}^{\infty} (-1)^n \dfrac{9\, x^n}{2^n}$

(d) $\sum_{n=0}^{\infty} (-1)^n \dfrac{2^{n+1}\, x^n}{3^{n-1}}$

(e) $\sum_{n=0}^{\infty} \dfrac{2^n \cdot x^n}{27}$

26. The Taylor series of $f(x) = e^x \cos(2x)$ about $x = 0$ is given by

(a) $1 + x - \dfrac{3}{2}x^2 - \dfrac{11}{6}x^3 + \dots$

(b) $1 - x + x^2 - \dfrac{3}{2}x^3 + \dots$

(c) $x + x^2 - \dfrac{3}{2}x^3 - \dfrac{7}{24}x^4 + \dots$

(d) $2 + x - x^2 - \dfrac{11}{6}x^3 + \dots$

(e) $1 - x - x^2 - x^3 + \dots$

27. The series $\sum_{n=1}^{\infty}\left(\frac{n}{n+e^n}\right)^{2n}$ is

(a) convergent by the root test

(b) divergent by the root test

(c) a series for which the root test is inconclusive

(d) a divergent geometric series

(e) divergent by the $n^{th}-$ term test for divergence.

28. Applying the ratio test to the series $\sum_{n=1}^{\infty}\frac{(n+1)!\,4^n}{(2n+1)!}$, with $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\rho$, we conclude that

(a) $\rho=0$ and the series is convergent

(b) $\rho=4$ and the series is divergent

(c) $\rho=\frac{1}{2}$ and the series is convergent

(d) $\rho=2$ and the series is convergent

(e) $\rho=1$ and the test is inconclusive.