King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 101
Thursday, January 27, 2011
Net Time Allowed: 180 minutes

MASTER VERSION

1. If $f(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ 1+x & \text{if } -1 < x < 3 \\ 4 & \text{if } x \geq 3 \end{cases}$,

then $\int_{-2}^{5} f(x)dx =$

(a) 16

(b) 12

(c) 8

(d) 4

(e) -10

2. $\int \left(\frac{2-x}{x}\right)^2 dx =$

(a) $x - \frac{4}{x} - 4\ln|x| + C$

(b) $x - \frac{4}{x^3} - 4\ln|x| + C$

(c) $\frac{1}{3}\left(\frac{2-x}{x}\right)^3 + C$

(d) $\frac{4}{x^2} - \frac{4}{x} + C$

(e) $2x - \frac{3}{x} + 4\ln|x| + C$

3. The average value of $f(x) = \dfrac{1}{(2x-1)^2}$ over the interval $[1, 3]$ is equal to

(a) $1/5$

(b) $-1/20$

(c) $3/5$

(d) $-7/2$

(e) $1/2$

4. If $F(x) = \displaystyle\int_3^{x^2} \frac{t}{t^3+1}\, dt$, then $F'(x) =$

(a) $\dfrac{2x^3}{x^6+1}$

(b) $\dfrac{x^2}{x^6+1}$

(c) $\dfrac{x^2}{x^6+1} - \dfrac{3}{28}$

(d) $\dfrac{x^2}{x^3+1}$

(e) $\dfrac{x}{x^3+1}$

5. If $\{S_n\}_{n=1}^{\infty}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt[3]{n}} - \frac{1}{\sqrt[3]{n+1}}\right)$, then $S_n =$

(a) $1 - \frac{1}{\sqrt[3]{n+1}}$

(b) $\frac{1}{\sqrt[3]{n}} - \frac{1}{\sqrt[3]{n+1}}$

(c) $\frac{n}{\sqrt[3]{n}}$

(d) $\frac{1}{\sqrt[3]{2}} - \frac{1}{\sqrt[3]{n+1}}$

(e) $1 - \frac{1}{\sqrt[3]{n}}$

6. The sequence $\left\{\frac{\ln(n^3)}{n}\right\}_{n=1}^{\infty}$

(a) is convergent and its limit is 0

(b) is convergent and its limit is 1

(c) is convergent and its limit is 3

(d) is convergent and its limit is $\frac{-1}{3}$

(e) is divergent

7. If f is continuous and $\int_0^2 f(x)dx = A$, then $\int_{1/3}^1 f(3x-1)dx =$

(a) $\dfrac{A}{3}$

(b) $3A$

(c) $3A - 1$

(d) $A + \dfrac{1}{3}$

(e) $\dfrac{2A}{3}$

8. The first three terms of the Taylor series of $f(x) = \dfrac{1}{\sqrt{x}}$ about $a = 4$ are given by

(a) $\dfrac{1}{2} - \dfrac{1}{16}(x-4) + \dfrac{3}{256}(x-4)^2$

(b) $\dfrac{1}{2} - \dfrac{1}{2}(x-4) + \dfrac{3}{4}(x-4)^2$

(c) $\dfrac{1}{2} - (x-4) + (x-4)^2$

(d) $\dfrac{1}{2} + \dfrac{1}{16}(x+4) - \dfrac{1}{128}(x+4)^2$

(e) $\dfrac{1}{4} - \dfrac{1}{16}(x-4) + \dfrac{3}{4}(x-4)^2$

9. If R_n is the Riemann sum for

$$f(x) = 2 - \frac{1}{3}\,x\,, \quad 1 \le x \le 4$$

with n rectangles and taking sample points to be the **right endpoints**, then $R_n =$

(a) $5 - \dfrac{3n+3}{2n}$

(b) $\dfrac{5}{3} - \dfrac{n+1}{n}$

(c) $5 - \dfrac{n+1}{2n}$

(d) $2 - \dfrac{n+1}{3n}$

(e) $5 - \dfrac{1}{n} + \dfrac{3n+1}{n^2}$

10. The series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6-3n^2}{2n^2+n}\right)^n$

(a) diverges by the root test

(b) converges by the root test

(c) converges by the ratio test

(d) diverges by the integral test

(e) is a series under which the root test is inconclusive

11. If $\dfrac{x}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$, then $A+B+C=$

(a) $\dfrac{1}{3}$

(b) $-\dfrac{1}{3}$

(c) 0

(d) 1

(e) $\dfrac{2}{3}$

12. The volume of the solid obtained by rotating the region bounded by the curves

$$y = \tan x,\ \ y = 0,\ \ x = 0,\ \ x = \frac{\pi}{3}$$

about the x-axis is

(a) $\pi\left(\sqrt{3} - \dfrac{\pi}{3}\right)$

(b) $\pi\left(1 - \dfrac{\pi}{2}\right)$

(c) $\pi\left(\dfrac{1}{\sqrt{3}} - \pi\right)$

(d) $\dfrac{\pi^2}{2}$

(e) $\sqrt{3} - \dfrac{\pi^2}{3}$

13. $\displaystyle\int \frac{\tan^2 x}{\csc x}\, dx =$

(a) $\sec x + \cos x + C$

(b) $\csc x - \sin x + C$

(c) $\dfrac{1}{3}\sin^3 x + C$

(d) $\cot^2 x + \cos x + C$

(e) $\dfrac{1}{2}\cos^2 x - \sin x + C$

14. The volume of the solid obtained by rotating the rergion bounded by the curves $y = x - x^2$ and $y = 0$ about the line $x = -1$ is equal to

(a) $\dfrac{\pi}{2}$

(b) $2\pi + 1$

(c) $\dfrac{6\pi}{5}$

(d) $\dfrac{\pi}{12}$

(e) $\pi - 1$

15. Using the binomial series, we get $\sqrt[3]{1+x} =$ (for $|x| < 1$)

(a) $1 + \frac{1}{3}\,x - \frac{1}{9}\,x^2 + \frac{5}{81}\,x^3 + \cdots$

(b) $1 + \frac{1}{3}\,x + \frac{1}{9}\,x^2 + \frac{1}{27}\,x^3 + \cdots$

(c) $1 + x - \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + \cdots$

(d) $1 - \frac{1}{3}\,x + \frac{1}{6}\,x^2 + \frac{3}{27}\,x^3 + \cdots$

(e) $1 + \frac{1}{3}\,x + \frac{1}{9}\,x^2 - \frac{5}{81}\,x^3 + \cdots$

16. The series $\sum_{n=1}^{\infty} (-1)^n \frac{3^{n-1}}{2^{2n+3}}$ is

(a) convergent and its sum is $-\frac{1}{56}$

(b) convergent and its sum is $-\frac{1}{24}$

(c) convergent and its sum is $\frac{3}{14}$

(d) convergent and its sum is $\frac{3}{28}$

(e) divergent

17. The smallest number of terms required to approximate the sum of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n^3 - 1}$ so that $|\text{error}| < 0.001$ is

(a) 10

(b) 8

(c) 20

(d) 100

(e) 12

18. The area of the surface obtained by rotating the curve

$$y = \tan^{-1} x \quad 0 \le x \le 1$$

about the y-axis is given by

(a) $\int_0^1 2\pi x \sqrt{1 + \frac{1}{(1+x^2)^2}}\, dx$

(b) $\int_0^1 2\pi \tan^{-1} x \sqrt{1 + \frac{1}{(1+x^2)^2}}\, dx$

(c) $\int_0^1 2\pi \tan^{-1} x \sqrt{1 + (\tan^{-1} x)^2}\, dx$

(d) $\int_0^{\pi/4} 2\pi y \sqrt{1 + \sec^4 y}\, dy$

(e) $\int_0^{\pi/4} 2\pi \tan y \sqrt{1 + \sec^2 y}\, dy$

19. $\displaystyle\int_0^{\ln 2} e^{x+2e^x}\,dx =$

(a) $\dfrac{1}{2}(e^4 - e^2)$

(b) e^2

(c) $2e^4$

(d) $e^4 - e^3$

(e) $\dfrac{1}{2}(e^2 - e)$

20. $\displaystyle\int \cos(\ln x)dx =$

(a) $\dfrac{x}{2}[\cos(\ln x) + \sin(\ln x)] + C$

(b) $2x\sin(\ln x) + C$

(c) $\dfrac{1}{2}\cos^2(\ln x) + C$

(d) $\sin(\ln x) - \dfrac{x}{x^2-1}\cos(\ln x) + C$

(e) $x[\cos(\ln x) + \sin(\ln x)] - [\cos(\ln x) - \sin(\ln x)] + C$

21. The interval of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-2)^n}{3n+1}$$

is

(a) $(1,3]$

(b) $(1,3)$

(c) $[1,3]$

(d) $(-1,1)$

(e) $(-1,3]$

22. The area of the region that lies below the graph of the curve $y=\dfrac{1}{x^2+10x+29}$ and above the x-axis for $x \geq -3$ is equal to

(a) $\dfrac{\pi}{8}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{3\pi}{4}$

(d) $\dfrac{1}{2}$

(e) ∞

23. The Maclaurin series for $f(x) = x^2 \cos(\sqrt{2}\, x)$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{(2n)!} x^{2n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (\sqrt{2})^n}{(2n)!} x^{2n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(\sqrt{2})^n}{(2n+1)!} x^{2n+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+2}}{2n} x^{2n+1}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (\sqrt{2}x)^{2n+2}}{(2n)!}$

24. $\displaystyle\int \frac{1}{3x + \sqrt[3]{x}}\, dx =$

(a) $\dfrac{1}{2} \ln(3\sqrt[3]{x^2} + 1) + C$

(b) $\ln|3x + \sqrt[3]{x}| + C$

(c) $\dfrac{1}{3} \ln|x| - \dfrac{3}{2}\, x^{2/3} + C$

(d) $\dfrac{1}{2} \ln|\sqrt[3]{x} + 3| + C$

(e) $\dfrac{1}{\sqrt{3}} \tan^{-1}\left(\dfrac{\sqrt[3]{x}}{3}\right) + C$

25. If $a_n = \dfrac{[1 \cdot 3 \cdot 5 \cdots (2n+1)]^2}{4^n \cdot (2n)!}$, then $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} =$

(a) $\dfrac{1}{4}$

(b) 4

(c) 0

(d) ∞

(e) $\dfrac{1}{2}$

26. Using the Maclaurin series for $f(x) = \dfrac{1}{(1-x)^2}$, we find that the sum of the series $\sum_{n=1}^{\infty} \dfrac{n}{5^n}$ is equal to

(a) $\dfrac{5}{16}$

(b) $\dfrac{1}{5}$

(c) $\dfrac{4}{5}$

(d) $\dfrac{2}{3}$

(e) $\dfrac{3}{8}$

27. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+2^n}$ is

(a) absolutely convergent

(b) conditionally convergent

(c) divergent

(d) divergent by the alternating series test

(e) convergent by the integral test

28. The length of the curve

$$y = \frac{1}{8}x^4 + \frac{1}{4x^2}, \quad 1 \le x \le 2$$

is equal to

(a) $\frac{33}{16}$

(b) $\frac{15}{8}$

(c) $\frac{2}{3}$

(d) 1

(e) $\frac{28}{15}$