King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
092
Saturday, June 12, 2010
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\sum_{n=1}^{\infty} \frac{1+(-2)^n}{3^n} =$

(a) 0.1

(b) 0.01

(c) 0.001

(d) 1.1

(e) 1.01

2. $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} x^3 \sin(x^2) dx =$

(a) $\frac{\pi - 1}{2}$

(b) $\frac{\pi + 1}{4}$

(c) $\frac{\pi}{2}$

(d) $\pi + 1$

(e) $\pi - 1$

3. $\int \cot^3 \alpha \csc^3 \alpha \, d\alpha =$

(a) $\frac{1}{3}\csc^3 \alpha - \frac{1}{5}\csc^5 \alpha + C$

(b) $\frac{1}{3}\csc^3 \alpha + \frac{1}{5}\csc^5 \alpha + C$

(c) $\frac{1}{2}\csc^2 \alpha + \frac{1}{4}\csc^4 \alpha + C$

(d) $\frac{1}{2}\csc^2 \alpha - \frac{1}{4}\csc^4 \alpha + C$

(e) $\frac{1}{4}\csc^4 \alpha + C$

4. The improper integral $\int_{-\infty}^{1} \frac{1}{2} e^{2x} \, dx$ is

(a) convergent and its value is $e^2/4$

(b) convergent and its value is $e^3/8$

(c) convergent and its value is $e/2$

(d) convergent and its value is e

(e) divergent

5. The length of the curve $x = \frac{2}{3} y^{3/2}$ from $y = 0$ to $y = 3$ is

(a) $\frac{14}{3}$

(b) $\frac{11}{3}$

(c) $\frac{17}{3}$

(d) 5

(e) 4

6. The set of all values of P, in interval notation, for which the series $\sum_{n=1}^{\infty} n(1+n^2)^P$ is convergent, is

(a) $(-\infty, -1)$

(b) $(0, \infty)$

(c) $(-\infty, 0)$

(d) $(1, \infty)$

(e) $(-\infty, 1)$

7. The interval on which the curve $y = \int_0^x \frac{1}{1+t+t^2}dt$ is concave upward is

(a) $\left(-\infty, -\frac{1}{2}\right)$

(b) $(-\infty, \infty)$

(c) $\left(\frac{1}{2}, \infty\right)$

(d) $(1, \infty)$

(e) $(-\infty, 1)$

8. If the region bounded by the curves $y = 4(x-1)^2$ and $y = 4$ is revolved about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^2 2\pi(8x - 4x^2)(x+1)\, dx$

(b) $\int_{-1}^2 2\pi(8x - 4x^2)(x+1)\, dx$

(c) $\int_0^2 16\pi(x-1)^4\, dx$

(d) $\int_0^2 8\pi(x-1)^2(x+1)\, dx$

(e) $\int_{-1}^2 8\pi(x-1)^2(x+1)\, dx$

9. The volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the line $y = 1$, is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) π

(e) $\frac{\pi}{4}$

10. The average value of the function $f(x) = \frac{1}{2}\sin x \sin 2x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$ is

(a) $\frac{\sqrt{2}+4}{9\pi}$

(b) $\frac{\sqrt{2}-4}{9\pi}$

(c) $\frac{\sqrt{2}+4}{3\pi}$

(d) $\frac{\sqrt{3}+4}{3\pi}$

(e) $\frac{2\sqrt{3}+4}{3\pi}$

11. $\displaystyle\int \frac{dx}{x(x^2+1)} =$

(a) $\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right) + C$

(b) $\ln\left(|x|\sqrt{x^2+1}\right) + C$

(c) $\ln\left(\dfrac{\sqrt{x^2+1}}{|x|}\right) + C$

(d) $\ln(|x|(x^2+1)) + C$

(e) $\ln\left(\dfrac{x^2+1}{|x|}\right) + C$

12. The area of the surface obtained by rotating the curve $y = \sqrt{x}$, $2 \le x \le 6$, about the x-axis, is equal to

(a) $\dfrac{49\pi}{3}$

(b) $\dfrac{79\pi}{3}$

(c) 49π

(d) 79π

(e) $\dfrac{101\pi}{6}$

13. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} 2^n n^p (x+1)^n, \text{ where } p \text{ is a constant,}$$

is

(a) $\frac{1}{2}$

(b) 2

(c) 1

(d) 4

(e) $\frac{1}{4}$

14. The series $\sum_{n=1}^{\infty} \frac{1+\sin^2 n}{n+n\sqrt{n}}$

(a) converges by the comparison test

(b) diverges by the integral test

(c) is a convergent geometric series

(d) is a convergent p-series

(e) diverges by the root test

15. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)} =$

(a) $\frac{11}{6}$

(b) $\frac{3}{2}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

(e) 1

16. The error of using the sum of the first 10 terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$ can be estimated as a number in the interval

(a) $(0, 0.1)$

(b) $(0.1, 0.2)$

(c) $(0.2, 0.3)$

(d) $(0.3, 0.6)$

(e) $(0.4, 0.5)$

17. The smallest number of terms, needed in order to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$ with |error| less than 0.001, is

(a) 100

(b) 50

(c) 10

(d) 150

(e) 200

18. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(a) converges conditionally

(b) converges absolutely

(c) diverges

(d) is a convergent geometric series

(e) is a divergent geometric series

19. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$ is

(a) $\left[-\frac{1}{2}, 0\right]$

(b) $\left[\frac{1}{4}, 1\right]$

(c) $\left(-\frac{1}{2}, 0\right)$

(d) $\left(\frac{1}{4}, 1\right]$

(e) $\left[-\frac{1}{2}, 0\right)$

20. The power series representation for the function $f(x) = \frac{x}{9+x^2}$ is

(a) $\frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n+1}$

(b) $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n}$

(c) $\frac{1}{3} \sum_{n=0}^{\infty} (-1)^{n-1} \left(\frac{x}{3}\right)^{2n+1}$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{9^n}$

(e) $\frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{9}\right)^{2n+1}$

21. The first three nonzero terms of the Taylor series of $f(x) = \sin(2x)$ about $a = \frac{\pi}{2}$ are given by

(a) $-2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^5$

(b) $1 - 2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3$

(c) $-2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^2 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^3$

(d) $2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^5$

(e) $-2\left(x - \frac{\pi}{2}\right) + 4\left(x - \frac{\pi}{2}\right)^2 - 4\left(x - \frac{\pi}{2}\right)^5$

22. If the Maclaurin series of $(1+x)^{3/2}$ is

$$A + Bx + Cx^2 + Dx^3 + Ex^4 + \cdots,$$

then $D + E =$

(a) $-\frac{5}{128}$

(b) $\frac{9}{128}$

(c) $\frac{7}{16}$

(d) $-\frac{7}{16}$

(e) $-\frac{7}{128}$

23. $\displaystyle\int_{-1}^{1} \frac{1+\tan x + x^2}{1+x^2}\, dx =$

(a) 2

(b) 0

(c) 1

(d) 3

(e) 4

24. The area of the triangle bounded by the lines

$$y = x,\ y = -3x \text{ and } y = -x + 2$$

is equal to

(a) 2

(b) 3

(c) 4

(d) $\frac{1}{2}$

(e) $\frac{4}{3}$

25. $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx =$

(a) $-\sqrt{3-2x-x^2} - \sin^{-1}\left(\frac{x+1}{2}\right) + C$

(b) $\sqrt{3-2x-x^2} + \cos^{-1}\left(\frac{x+1}{2}\right) + C$

(c) $-\sqrt{3-2x-x^2} + \sin\left(\frac{x+1}{2}\right) + C$

(d) $\sqrt{3-2x+x^2} + \cos^{-1}\left(\frac{x+1}{2}\right) + C$

(e) $\sqrt{3-2x-x^2} + C$

26. The improper integral $\int_0^3 \frac{3\,dx}{x^2-5x+4}$ is

(a) divergent

(b) convergent and its value is $\ln 4$

(c) convergent and its value is $\ln 3$

(d) convergent and its value is $\ln 2$

(e) convergent and its value is 0

27. The limit of the sequence defined by $s_n = \frac{1}{n}\sin\left(\frac{n\pi}{4}\right) + \frac{(2n+6)}{(n+1)}$

(a) is equal to 2

(b) is equal to 0

(c) is equal to 1

(d) oscillates between -1 and 1

(e) is ∞

28. The series $\sum_{n=1}^{\infty}(-1)^n\frac{n!+n}{(n+1)!}$

(a) converges conditionally

(b) converges absolutely

(c) diverges by alternating series test

(d) diverges because $\sum_{n=1}^{\infty}\frac{n!+n}{(n+1)!}$ diverges

(e) converges because $\sum_{n=1}^{\infty}\frac{n!+n}{(n+1)!}$ converges