King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 122
Tuesday 21/05/2013
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\int_1^e \frac{1}{x} \cdot \frac{\ln x}{1+(\ln x)^2}\,dx =$

(a) $\ln\sqrt{2}$

(b) $\ln 4$

(c) $\frac{1}{2}$

(d) $\frac{1}{2} + \ln\sqrt{2}$

(e) $\ln\sqrt{3}$

2. The area of the region bounded by the curve $y = \frac{6}{x}$ and the line $y = -x + 5$ from $x = 1$ to $x = 3$ is given by

(a) $\int_1^2 \frac{x^2 - 5x + 6}{x}\,dx + \int_2^3 \frac{-x^2 + 5x - 6}{x}\,dx$

(b) $\int_1^3 \frac{-x^2 + 5x - 6}{x}\,dx$

(c) $\int_1^2 \frac{-x^2 + 5x + 6}{x}\,dx + \int_2^3 \frac{x^2 - 5x + 6}{x}\,dx$

(d) $\int_1^3 \frac{x^2 - 5x + 6}{x}\,dx$

(e) $\int_1^2 \frac{6}{x}\,dx - \int_2^3 (-x + 5)\,dx$

3. The volume of the solid generated by revolving the region bounded by the parabola $y = -x^2 + 4$ and the line $x - y + 2 = 0$, about the line $y = -4$, is given by the definite integral

(a) $\int_{-2}^{1} \pi(x^4 - 17x^2 - 12x + 28)\, dx$

(b) $\int_{-2}^{1} \pi(x^4 + 2x^3 - 4x^2 - 4x - 12)\, dx$

(c) $\int_{-2}^{1} \pi(x^4 + 18x^2 + 14x - 28)\, dx$

(d) $\int_{-2}^{1} \pi(x^4 + 18x^2 + 14x - 28)\, dx$

(e) $\int_{-2}^{1} \pi(x^4 - 19x^2 + 12x + 28)\, dx$

4. If the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = b$ is equal to $\frac{14}{3}$, then $b =$

(a) 3

(b) 2

(c) 1

(d) 0

(e) 4

5. $\int x(\ln(2x))^2\,dx =$

(a) $\frac{1}{2}(x\ln(2x))^2 - \frac{1}{2}x^2\ln(2x) + \frac{1}{4}x^2 + c$

(b) $\frac{1}{2}(x\ln(2x))^2 + \frac{1}{2}x^2\ln(2x) + x^2 + c$

(c) $\frac{(\ln(2x))^3}{3} + x + c$

(d) $\frac{(\ln(2x))^3}{3} - x + c$

(e) $(x\ln(2x))^2 + \frac{1}{4}x^2 - \ln(2x) + c$

6. $\int 4\tan^3 x\,dx =$

(a) $2\tan^2 x + 4\ln|\cos x| + c$

(b) $2\tan^2 x + \ln|\cos x| + c$

(c) $\tan^4 x + c$

(d) $2\tan^2 x + \cot x\,\cos^2 x + c$

(e) $-4\tan^2 x + \ln|\cos x| + c$

7. $\int x\sqrt{1-x^4}\,dx =$

(a) $\frac{1}{4}(x^2\sqrt{1-x^4}+\sin^{-1}(x^2))+c$

(b) $\frac{1}{4}(x^2\sqrt{1-x^4}-3\sin^{-1}(x^2))+c$

(c) $x+x^2\sqrt{1-x^4}+c$

(d) $\sqrt{1-x^4}+\sin^{-1}(x^2)+c$

(e) $\frac{1}{2}\sqrt{1-x^4}+2\sin^{-1}(x^2)+c$

8. $\int \frac{3x^3-3x^2+4}{x^2-x}\,dx =$

(a) $\frac{3}{2}x^2+4\ln\left|\frac{x-1}{x}\right|+c$

(b) $\frac{3}{2}x^2+2\ln|x^2-x|+c$

(c) $\frac{3}{2}x^2+8\ln\left|\frac{x}{x-1}\right|+c$

(d) $3x^2+2\ln\left|\frac{x-1}{x}\right|+c$

(e) $3x^2+2\ln|x^2-x|+c$

9. $\int (x^2+1)\,\text{sech}(\ln x)\,dx =$

(a) $x^2 + c$

(b) $x^2 \ln x + \tanh(\ln x) + c$

(c) $\left(\dfrac{x^3}{3} + x\right)\text{sech}(\ln x) + c$

(d) $\text{sech}(\ln x) + x^2\text{sech}(\ln x) + c$

(e) $x^3 + c$

10. The area of the surface generated by revolving the curve $y = \dfrac{x^3}{3}$, $0 \le x \le 1$, about the x-axis is equal to

(a) $\pi\left(\dfrac{\sqrt{8}-1}{9}\right)$

(b) $\dfrac{2\pi\sqrt{3}}{9}$

(c) $2\,\pi$

(d) $\pi(\sqrt{7}-1)$

(e) $2\pi\left(\dfrac{\sqrt{8}-2}{9}\right)$

11. If $f(x) = \int_{e^x}^{1} \sin(\ln t)\, dt$, then $f'\left(\frac{\pi}{2}\right) =$

(a) $-e^{\frac{\pi}{2}}$

(b) 0

(c) -1

(d) $\sin(\ln 2)$

(e) $-e^{\frac{\pi}{2}} \sin\left(\ln \frac{\pi}{2}\right)$

12. Express $\int e^{x^2}\, dx$ as a power series

(a) $c + x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \dots$

(b) $c + x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots$

(c) $c + 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$

(d) $c + 1 - x^2 + x^3 - x^4 + \dots$

(e) $c + x^2 + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$

13. If A, B and C are the undetermined coefficients of the partial fractions decomposition of the rational function $\dfrac{x}{x^3-1}$, then $A^2+B^2+C^2$ is equal to

(a) $\dfrac{1}{3}$

(b) $\dfrac{1}{9}$

(c) $\dfrac{2}{3}$

(d) $\dfrac{2}{9}$

(e) 1

14. If f and h are integrable functions such that $\int_1^9 f(x)dx = -1$, $\int_7^9 f(x)dx = 5$ and

$\int_1^7 h(x)dx = 4$, then $\int_7^1 [h(x) - f(x)]dx =$

(a) -10

(b) 8

(c) 6

(d) -8

(e) 12

15. The improper integral $\int_0^{3\pi/2} \frac{\sin x}{1+\cos x}\, dx$ is

(a) divergent

(b) covergent and its value is $\ln \frac{1}{2}$

(c) covergent and its value is $\ln 2$

(d) covergent and its value is 0

(e) covergent and its value is $\frac{1}{2}$

16. $\int \frac{\sin^{-1}(e^{-x})}{\sqrt{e^{2x}-1}}\, dx =$

(a) $-\frac{1}{2}(\sin^{-1}(e^{-x}))^2 + c$

(b) $e^{-x}\sin^{-1}(e^{-x}) + c$

(c) $-\frac{3}{2}(\sin^{-1}(e^{-x}))^2 + c$

(d) $-\frac{1}{2}e^{-x}(\sin^{-1}(e^{-x}))^2 + c$

(e) $(\sin^{-1}(e^{-x}))^2 + c$

17. The sequence $\{2n - \sqrt{4n^2 - n}\}$ is

(a) converges to $\frac{1}{4}$

(b) converges to 1

(c) converges to 0

(d) converges to $\frac{1}{2}$

(e) diverges

18. The series

$$\frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \frac{1}{(5)(6)} + \frac{1}{(6)(7)} + \dots$$

is

(a) convergent and its sum is $\frac{1}{3}$

(b) convergent and its sum is 0

(c) convergent and its sum is $\frac{3}{4}$

(d) convergent and its sum is $\frac{1}{5}$

(e) divergent

19. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3^{n-1}}{5^{n+1}} =$

(a) 0.025

(b) 0.01

(c) 0.5

(d) 0.005

(e) 2.5

20. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^3}$ is

(a) convergent by the Comparison Test

(b) divergent by the nth-Term Test for Divergence

(c) divergent by the Ratio Test

(d) divergent by the Integral Test

(e) convergent by the Ratio Test

21. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}+\sqrt{n+3}}$

(a) converges conditionally

(b) diverges by the Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$

(c) converges absolutely

(d) diverges by the Ratio Test

(e) diverges by the nth-Term Test for Divergence.

22. The series $\sum_{n=1}^{\infty} \frac{3 \cdot 2^{2n}}{3^{n+1} n^n}$ is

(a) convergent by the Root Test

(b) divergent by the Root Test

(c) a series for which the Root Test is inconclusive

(d) a divergent geometric series

(e) a convergent p-series.

23. The series $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^3}{(3n+1)!}$ is

(a) absolutely convergent

(b) conditionally convergent

(c) divergent by the Ratio test

(d) a divergent p-series.

(e) a series for which the Ratio test is inconclusive

24. The interval of convergence I and the radius of convergence R of the series $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n} (x-3)^n$, are given by

(a) $I = (-6, 12)\,,\ R = 9$

(b) $I = (-3, 3)\,,\ R = 3$

(c) $I = (-6, 12]\,,\ R = 6$

(d) $I = (-3, 3)\,,\ R = 6$

(e) $I = [-6, 12)\,,\ R = 9$.

25. The Taylor polynomial of order 3 generated by $f(x) = \ln(2+x)$ at $a = -1$ is

(a) $P_3(x) = (x+1) - \frac{1}{2}(x+1)^2 + \frac{1}{3}(x+1)^3$

(b) $P_3(x) = 1 + (x+1) + \frac{1}{2}(x+1)^2 - \frac{1}{3}(x+1)^3$

(c) $P_3(x) = (x-1) + \frac{1}{2}(x-1)^2 - (x+1)^3$

(d) $P_3(x) = (x+1) + \frac{1}{2}(x+1)^2 + \frac{1}{6}(x+1)^3$

(e) $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

26. let $f(x) = \frac{1}{2-x}$, $|x| < 2$, then the power series representation of $f''(x)$ is

(a) $\sum_{n=2}^{\infty} \frac{n(n-1)x^{n-2}}{2^{n+1}}$

(b) $\sum_{n=2}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-1}$

(c) $\sum_{n=0}^{\infty} \frac{n(n-1)x^{n-2}}{2^{n}}$

(d) $\sum_{n=2}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-2}$

(e) $\sum_{n=3}^{\infty} n(n-1)\left(\frac{x}{2}\right)^{n-3}$

27. The coefficient of x^5 in the product of the Maclaurin series of $\sin x$ and $\dfrac{1}{1-x}$ is equal to

(a) $\dfrac{101}{120}$

(b) $\dfrac{97}{120}$

(c) $\dfrac{17}{60}$

(d) $\dfrac{131}{120}$

(e) $\dfrac{1}{120}$

28. For $-1 < x < 1$, the Maclaurian series generated by $f(x) = \sqrt[3]{(1-x)^2}$ is

(a) $1 - \dfrac{2}{3}x - \dfrac{x^2}{9} - \dfrac{4x^3}{81} + \ldots$

(b) $1 - \dfrac{2}{3}x + \dfrac{x^2}{6} - \dfrac{4x^3}{81} + \ldots$

(c) $1 - \dfrac{2}{3}x - \dfrac{x^2}{9} + \dfrac{4x^3}{27} + \ldots$

(d) $1 - \dfrac{2}{3}x + \dfrac{x^2}{9} + \dfrac{4x^3}{63} + \ldots$

(e) $1 + \dfrac{2}{3}x - \dfrac{x^2}{6} - \dfrac{4}{81}x^3 + \ldots$