King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 101
Wednesday, November 3, 2010
Net Time Allowed: 120 minutes

MASTER VERSION

1. The area under the graph of $f(x) = \dfrac{1}{x+1}$ from $x = 1$ to $x = 3$ using three rectangles and left endpoints is approximately equal to

(a) $47/60$

(b) $33/25$

(c) $27/20$

(d) $49/50$

(e) $25/38$

2. $\int (x+2)\tan(x^2+4x)\,dx =$

(a) $\dfrac{1}{2}\ln|\sec(x^2+4x)| + C$

(b) $\dfrac{1}{2}\cot(x^2+4x) + C$

(c) $\ln|\sec(x^2+4x)| + C$

(d) $2\ln|\sin(x^2+4x)| + C$

(e) $\dfrac{1}{2}\ln|\csc(x^2+4x)| + C$

3. If f' is continuous, $f(8) = 10$, and $\int_2^8 f'(x)dx = 8$, then $f(2) =$

(a) 2

(b) 10

(c) 18

(d) 4

(e) 8

4. If $g(x) = \int_x^2 \ln\left(\frac{2}{t}\right) dt$, then $g'(x) =$

(a) $\ln\left(\frac{x}{2}\right)$

(b) $\ln\left(\frac{2}{x}\right)$

(c) $\ln x$

(d) $-2\ln x$

(e) $\frac{\ln x}{x}$

5. $\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{\pi}{n}\left(1+\sin\left(\frac{3\pi i}{n}\right)\right)^2 =$

(a) $\int_0^{\pi} (1+\sin(3x))^2 \, dx$

(b) $\int_0^{\pi} (1+\sin(3\pi x))^2 \, dx$

(c) $\int_{\pi}^{2\pi} 2(1+\sin(3x))^2 \, dx$

(d) $\int_0^{2\pi} (1+\sin(3\pi x))^2 \, dx$

(e) $\int_0^{3\pi} (1+\sin x)^2 \, dx$

6. $\int_1^{-2} f(x)dx - \int_3^{-2} f(x)dx - \int_1^{3} f(x)dx =$

(a) 0

(b) $\int_{-2}^{1} f(x)dx$

(c) $\int_{-2}^{3} f(x)dx$

(d) $\int_3^{1} f(x)dx$

(e) $2\int_1^{3} f(x)dx$

7. The volume of the solid obtained by rotating the region bounded by the curves

$$y = e^x, \quad y = 1, \; x = 2$$

about the x-axis is equal to

(a) $\frac{\pi}{2}(e^4 - 5)$

(b) $\frac{\pi}{2}(e^2 - 1)$

(c) πe^4

(d) $\pi \left(\frac{e^2}{2} - 3 \right)$

(e) $\pi(e^2 + 2)$

8. $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{i^5}{n^6} =$

(a) $\frac{1}{6}$

(b) $\frac{5}{6}$

(c) $\frac{2}{3}$

(d) $\frac{1}{4}$

(e) $\frac{3}{5}$

9. If $G(x) = \int_2^x g(t)dt$, where $g(t) = \int_2^{2\sqrt{t}} \frac{\sqrt{9+w^2}}{1+2w^2}\, dw$, then $G''(4) =$

(a) $\frac{5}{66}$

(b) $\frac{15}{33}$

(c) $\frac{19}{34}$

(d) $\frac{5}{28}$

(e) $\frac{19}{66}$

10. The area of the region enclosed by the line $x = 0$ and the parabola $x = y^2 - 2y$ is equal to

(a) $\frac{4}{3}$

(b) $-\frac{8}{3}$

(c) $\frac{5}{2}$

(d) $\frac{7}{2}$

(e) $\frac{15}{4}$

11. $\displaystyle\int_{-\pi}^{\pi} \frac{t+\sin t}{2+\cos t}\,dt =$

(a) 0

(b) $\dfrac{\pi^3}{2}$

(c) $\dfrac{\pi^4}{3} - 1$

(d) $2\ln 3$

(e) $\pi \ln 2$

12. $\displaystyle\int_{0}^{3} x\sqrt{81-x^4}\,dx =$

(a) $\dfrac{81\pi}{8}$

(b) $\dfrac{9\pi}{8}$

(c) $\dfrac{27\pi}{2}$

(d) $\dfrac{81\pi}{2}$

(e) $\dfrac{10\pi}{7}$

13. The volume of the solid generated by revolving the region bounded by the curves

$$y = \frac{1}{x^2+1},\ y = 0,\ x = 0,\ x = 3$$

about the y-axis is equal to

(a) $\pi \ln 10$

(b) $2\pi \ln 5$

(c) $\frac{1}{2}\pi \ln 3$

(d) $\pi \ln 12$

(e) $3\pi \ln 3$

14. The area of the region that lies in **the first quadrant** and is enclosed by the curves

$$y = \frac{1}{x},\ y = x\,,\ y = 4x$$

is equal to

(a) $\ln 2$

(b) $\frac{1}{2} + \ln 2$

(c) $\frac{3}{8} - \ln 2$

(d) 1

(e) $\frac{3}{2} + \ln 2$

15. $\int x\sqrt[3]{2x-1}\,dx =$

(a) $\frac{3}{28}(2x-1)^{7/3}+\frac{3}{16}(2x-1)^{4/3}+C$

(b) $\frac{3}{14}(2x-1)^{7/3}+\frac{3}{8}(2x-1)^{4/3}+C$

(c) $\frac{1}{4}(2x-1)^{7/3}+\frac{1}{4}(2x-1)^{4/3}+C$

(d) $\frac{1}{4}(2x-1)^{4/3}+\frac{1}{4}(2x-1)^{1/3}+C$

(e) $\frac{3}{16}x^2(2x-1)^{4/3}+C$

16. The volume of the solid generated by rotating the region bounded by the curves

$$y=x^2,\ y=2-x,\ y=0$$

about the line $x=2$ is given by

(a) $\pi\int_0^1[(2-\sqrt{y})^2-y^2]dy$

(b) $\pi\int_0^2[(2-x^2)^2-x^2]dx$

(c) $\pi\int_0^2[(x^2)^2-(2-x)^2]dx$

(d) $\pi\int_0^1[(2-y)^2-(\sqrt{y})^2]dy$

(e) $\pi\int_0^1 x^4\,dx+\pi\int_1^2(2-x)^2dx$

17. The base of a solid is the triangular region with vertices $(0,0),(2,2),(0,2)$. If the cross sections of the solid perpendicular to the x-axis are **semicircles**, then the volume of the solid is equal to

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{8}$

(c) $\frac{3\pi}{8}$

(d) $\frac{\pi}{4}$

(e) $\frac{2\pi}{3}$

18. $\int_1^2 \frac{1}{x^2}\sqrt{\frac{x-1}{x}}\,dx =$

(a) $\frac{1}{3\sqrt{2}}$

(b) $\frac{\sqrt{3}}{2}$

(c) 1

(d) $\frac{2}{\sqrt{5}}$

(e) $7\sqrt{2}$

19. Which one of the following statements is **TRUE** about the integral $I = \int_0^{\pi/2} e^{\sin x}\, dx$

(a) $\frac{\pi}{2} \leq I \leq \frac{\pi e}{2}$

(b) $\frac{5e}{2} \leq I \leq 5e$

(c) $2\pi \leq I \leq 4\pi$

(d) $\pi e \leq I \leq 2\pi e$

(e) $\pi^2 \leq I \leq \pi^2 e$

20. The velocity (in m/s) of a particle moving along a line is given by

$$v(t) = t^2 - t - 2.$$

The distance traveled by the particle during the time interval $1 \leq t \leq 3$ is

(a) $3\,\text{m}$

(b) $4\,\text{m}$

(c) $1\,\text{m}$

(d) $\frac{2}{3}\,\text{m}$

(e) $\frac{1}{2}\,\text{m}$