King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

**Math 102**
**Exam I**
**093**
**Monday 19/07/2010**
**Net Time Allowed: 120 minutes**

# MASTER VERSION

1. To estimate the area under the graph of $f(x) = x\sin x$ from $x = 0$ to $x = \pi$ using four rectangles and the right endpoints we get

(a) $\dfrac{\pi^2(1+\sqrt{2})}{8}$

(b) $\dfrac{\pi^2(2-\sqrt{2})}{4}$

(c) $\dfrac{\pi^2(\sqrt{2}-1)}{8}$

(d) $\dfrac{\pi^2(2-\sqrt{2})}{8}$

(e) $\dfrac{\sqrt{2}\pi^2}{8}$

2. $\displaystyle\int \frac{\sqrt{x}+\sqrt[4]{x}}{\sqrt[3]{x}}\,dx =$

(a) $\frac{6}{7}x^{7/6} + \frac{12}{11}x^{11/12} + c$

(b) $-\frac{1}{x} + \frac{1}{2}x^2 + c$

(c) $42x^{7/6} + 132x^{11/12} + c$

(d) $\ln|x| + \frac{1}{2}x^2 + c$

(e) $\frac{6}{5}x^{5/6} + \frac{12}{19}x^{19/12} + c$

3. $\int_0^1 \frac{3x^3 + x^2 - 18x - 6}{3x+1} dx =$

(a) $-\frac{17}{3}$

(b) $-\frac{19}{3}$

(c) $\frac{14}{3}$

(d) $-\frac{11}{3}$

(e) $\frac{5}{3}$

4. $\int (\tan x + \cot x) dx =$

(a) $\ln|\tan x| + c$

(b) $\ln|\cot x| + c$

(c) $\ln|\sec x| + c$

(d) $\ln|\csc x| + c$

(e) $\ln|\sec x + \csc x| + c$

5. If $v(t) = (t-3)$ m/s is the velocity of a particle moving on a line at time $t$ in seconds, then the total distance traveled by the particle during the time interval $[0, 4]$ is

(a) $5m$

(b) $9m$

(c) $6m$

(d) $3m$

(e) $14m$

6. $\int \frac{\sin^{-1}\left(\frac{x}{2}\right)}{\sqrt{4-x^2}}\,dx =$

(a) $\frac{1}{2}\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 + c$

(b) $\ln\left|\sin^{-1}\left(\frac{x}{2}\right)\right| + c$

(c) $4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 + c$

(d) $\frac{1}{4-x^2} + c$

(e) $\sqrt{4-x^2}\sin^{-1}\left(\frac{x}{2}\right) + c$

7. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(3 - \frac{2i}{n}\right)^2 =$

(a) $\frac{13}{3}$

(b) $\frac{22}{3}$

(c) $\frac{14}{3}$

(d) $\frac{52}{3}$

(e) $0$

8. The area of the region enclosed by the curves $y = \sqrt{x}$, $y = x$, $x = 0$, and $x = 4$ is equal to

(a) $3$

(b) $\frac{3}{2}$

(c) $5$

(d) $\frac{5}{2}$

(e) $\frac{10}{3}$

9. If $\int_4^7 f(x)dx = 5$, then $\int_1^4 \frac{f(3\sqrt{x}+1)}{\sqrt{x}}\,dx =$

(a) $\frac{10}{3}$

(b) 30

(c) $\frac{35}{3}$

(d) 20

(e) $\frac{25}{3}$

10. The area of the region enclosed by $2x+y^2=3$ and $x-y=0$ is

(a) $\left[\frac{3}{2}y - \frac{1}{2}y^2 - \frac{1}{6}y^3\right]_{-3}^{1}$

(b) $\left[\frac{1}{2}y - \frac{3}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^{3}$

(c) $\left[\frac{3}{2}y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-3}^{1}$

(d) $\left[\frac{3}{2}y - \frac{1}{3}y^2 - \frac{1}{3}y^3\right]_{-3}^{1}$

(e) $\left[\frac{3}{2}y - \frac{2}{3}y^2 - \frac{1}{6}y^3\right]_{-1}^{3}$

11. The base of a solid is the region bounded by the parabola $y = 1 - x^2$ and the $x-$ axis. Each cross section perpendicular to the $x-$ axis is a square. The volume of the solid is

(a) $\frac{16}{15}$

(b) $\frac{4}{3}$

(c) $\frac{2}{3}$

(d) $\frac{15}{12}$

(e) $\frac{8}{15}$

12. The volume of the solid obtained by rotating the region bounded by the graphs of $y = \tan x, x = \frac{\pi}{4}$ and $y = 0$ about the $x-$ axis is equal to

(a) $\frac{4\pi - \pi^2}{4}$

(b) $\pi$

(c) $2\pi^2 - 4\pi$

(d) $\frac{3\pi^2 - 4\pi}{4}$

(e) $\frac{\pi}{4}$

13. $\int_{-1}^{0}(x+1)\ e^{-x(x+2)}\,dx =$

(a) $\frac{1}{2}(e-1)$

(b) $\frac{3}{2}e$

(c) $\frac{1}{4}(e+1)$

(d) $\frac{1}{4}(e-1)$

(e) $1-e$

14. An expression for the area under the graph of $f(x) = 3x^2 + 5x$, $0 \le x \le 2$, as a limit and using right endpoints is

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{24i^2}{n^3} + \frac{20i}{n^2}\right]$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{6i^2}{n^3} + \frac{10i}{n^2}\right]$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{3i^2}{n^3} + \frac{20i}{n^2}\right]$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{24i^2}{n^3} + \frac{5i}{n^2}\right]$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{3i^2}{4n^3} + \frac{5i}{2n^2}\right]$

15. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\csc x)(3\sin 2x + 5\sin x)dx =$

(a) $3 + \frac{5\pi}{3}$

(b) $1 + \frac{\pi}{3}$

(c) $3 + \frac{2\pi}{3}$

(d) $1 + \frac{5\pi}{3}$

(e) $5\pi$

16. If the plane region enclosed by the graphs of $y = x$ and $y = x^2$ is revolved about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^1 \pi(2\sqrt{y} - y - y^2)dy$

(b) $\int_{-1}^1 \pi(y - y^2)dy$

(c) $\int_0^1 \pi(4\sqrt{y} + 6y - y^2)dy$

(d) $\int_{-1}^1 \pi(y - y^2 - 2)dy$

(e) $\int_0^1 \pi(2\sqrt{y} - 6y - y^2)dy$

17. The equation of the tangent line to the graph of $f(x) = \int_{\sqrt{x}}^{x^3} e^{u^2} du$ at $x = 1$ is

(a) $y = \frac{5e}{2}x - \frac{5e}{2}$

(b) $y = \frac{3e}{2}x - \frac{3e}{2}$

(c) $y = \frac{2e}{3}x - \frac{2e}{3}$

(d) $y = \frac{5e}{2}x - \frac{e}{2}$

(e) $y = \frac{5e}{2}x + \frac{e}{2}$

18. If $k = \int_0^{\frac{\pi}{2}} e^{-\sin x} dx$, then

(a) $\frac{\pi}{2e} \leq k \leq \frac{\pi}{2}$

(b) $0 \leq k \leq \frac{\pi}{2e}$

(c) $k \geq \frac{\pi}{2}$

(d) $k \geq \frac{\pi}{2e} + \frac{\pi}{2}$

(e) $-\frac{\pi}{2e} \leq k \leq \frac{\pi}{2e}$

19. The value of $\int_{-\pi}^{\pi}(4+3\sin x)\sqrt{\pi^2-x^2}\,dx$ is equal to :
(Hint : write the integral as a sum of two integrals and interpreting one of these integrals in terms of an area)

(a) $2\pi^3$

(b) $0$

(c) $\frac{\pi^3}{2}$

(d) $8\pi^3$

(e) $\frac{\pi^3}{4}$

20. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{1}{n}+\frac{i}{n^2}+\frac{1}{n}e^{1+\frac{i}{n}}\right]=$

(a) $\int_1^2(x+e^x)\,dx$

(b) $\int_1^2(1+x+e^x)\,dx$

(c) $\int_0^1(x+e^x)\,dx$

(d) $\int_0^1(1+x+e^x)\,dx$

(e) $\int_1^2(x+x^2e^x)\,dx$