King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Fall
Wednesday 31/03/2010
Net Time Allowed: 120 minutes

MASTER VERSION

1. $\int \frac{e^{2x}}{1+e^{4x}}\,dx =$

(a) $\frac{1}{2}\tan^{-1}(e^{2x}) + c$

(b) $\tan^{-1}(e^{2x}) + c$

(c) $\frac{1}{4}\tan^{-1}(e^{2x}) + c$

(d) $\frac{1}{2}\tan^{-1}(e^{4x}) + c$

(e) $\tan^{-1}(e^{4x}) + c$

2. Using four rectangles and right endpoints, the area under the graph of

$$f(x) = \sin x$$

from $x = 0$ to $x = \pi$ is approximately equal to

(a) $\frac{\pi(1+\sqrt{2})}{4}$

(b) $\frac{\sqrt{2}(1+\pi)}{4}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi(1-\sqrt{2})}{2}$

(e) $\pi(1+\sqrt{2})$

3. $\int_0^{\frac{1}{2}} \left(\frac{6}{\sqrt{1-t^2}} + \frac{12t-2}{3\sqrt{t}} \right) dt =$

(a) π

(b) $\pi + \sqrt{2}$

(c) $\pi + 2\sqrt{2}$

(d) $\pi + 3\sqrt{2}$

(e) $\pi + 4\sqrt{2}$

4. $\int \frac{(x-2)^3}{x^2}\, dx =$

(a) $\frac{x^2}{2} - 6x + 12 \ln|x| + \frac{8}{x} + c$

(b) $\frac{x^2}{2} + 6x + 12 \ln|x| - \frac{8}{x} + c$

(c) $\frac{x^2}{2} - 6x + 12 \ln|x| - \frac{8}{x} + c$

(d) $\frac{x^2}{2} - 6x + 6 \ln|x| - \frac{4}{x} + c$

(e) $\frac{x^2}{2} + 6x - 12 \ln|x| + \frac{8}{x} + c$

5. If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, then $F''(2) =$

(a) $\sqrt{257}$

(b) $\sqrt{255}$

(c) $\sqrt{253}$

(d) $\sqrt{259}$

(e) $\sqrt{261}$

6. The volume of the solid resulting from the region: $y = -x^2 + 6x - 8\,;$ $y = 0$ which has been rotated about the $y-$ axis is given by the definite integral:

(a) $\int_2^4 2\pi\, x \left[-x^2 + 6x - 8\right]\, dx$

(b) $\int_2^4 \pi\, x \left[-x^2 + 6x - 8\right]\, dx$

(c) $\int_0^8 2\pi\, x \left[-x^2 + 6x - 8\right]\, dx$

(d) $\int_2^4 2\pi\, \left[-x^2 + 6x - 8\right]\, dx$

(e) $\int_0^4 2\pi\, x \left[-x^2 + 6x - 8\right]\, dx$

7. If f is an even function such that $\int_{-1}^{1} f(t)\,dt = 5$ and $\int_{-2}^{2} f(t)\,dt = 2$, then $\int_1^2 f(t)\,dt =$

(a) $-\frac{3}{2}$

(b) $\frac{3}{2}$

(c) 3

(d) -3

(e) 0

8. $\int_{-3}^{0} \left(|x-1| + \sqrt{9-x^2} \right) dx =$
(Hint: You may interpret the integral as an area)

(a) $\dfrac{9\pi + 30}{4}$

(b) $\dfrac{9\pi + 26}{4}$

(c) $\dfrac{9\pi + 34}{4}$

(d) $\dfrac{7\pi + 30}{4}$

(e) $\dfrac{7\pi + 34}{4}$

9. $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \frac{1}{1+\left(\frac{i-1}{n}\right)^2} =$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) 0

(d) 1

(e) 2

10. $\int \sin^2 x \, dx =$

(a) $\frac{x}{2} - \frac{\sin\, 2x}{4} + c$

(b) $-\cos\, x + c$

(c) $\frac{1}{2} \cos^2 x + c$

(d) $\frac{1}{2} \cos\, 2x + c$

(e) $\frac{x}{2} - \cos\, x + c$

11. If the region enclosed by the curves $y = x$ and $y = x^3$, where $x \geq 0$, is rotated about the $x-$axis, then the volume of the solid obtained is equal to

(a) $\frac{4\pi}{21}$

(b) $\frac{\pi}{4}$

(c) $\frac{11\pi}{21}$

(d) $\frac{7\pi}{21}$

(e) $\frac{\pi}{7}$

12. The area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$ is equal to

(a) $2\sqrt{2} - 2$

(b) $4\sqrt{2} + 2$

(c) $2\sqrt{2} + 2$

(d) 4

(e) $\sqrt{2} - 1$

13. The area of the region enclosed by the curves, $y = x^2 - 4\,, y = -2x + 4\,,$ and $y = -4$ is equal to

(a) $\frac{20}{3}$

(b) $\frac{17}{3}$

(c) $\frac{8}{5}$

(d) $\frac{12}{5}$

(e) 3

14. A particle moves along a line so that its velocity at time t is $v(t) = t - t^2$. The distance traveled by the particle during the time period $0 \leq t \leq 2$ is:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

15. The region whose area is equal to $\lim_{n\to\infty} \frac{4}{3}\left(\frac{\pi+3}{n}\right)\sum_{i=1}^{n} \sin\left(\frac{\pi\, i+3i-3n}{3n}\right)^2$ is the region

(a) under the graph of $y = 4\sin x^2$ from -1 to $\frac{\pi}{3}$.

(b) under the graph of $y = \sin x^2$ from 1 to $\frac{\pi}{3}$.

(c) under the graph of $y = \sin\left(\frac{x^2}{4}\right)$ from -1 to $\frac{\pi}{4}$.

(d) under the graph of $y = 4\sin x^2$ from 1 to $\frac{\pi}{4}$.

(e) under the graph of $y = \frac{4}{3}\sin x^2$ from 3 to π.

16. The slope of the line tangent to the curve $g(x) = \int_0^{x^3} \sqrt{t+e^t}\,dt$ at $x = 2$ is

(a) $12\sqrt{8+e^8}$

(b) $8\sqrt{8+e^8}$

(c) $8\sqrt{2+e^2}$

(d) $12\sqrt{2+e^2}$

(e) $12\sqrt{8+e^2}$

17. $\int_0^{\frac{\pi}{3}} \sin x \cos 2x \, dx =$

(a) $\frac{1}{12}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) $\frac{1}{6}$

18. Using cylindrical shells, the volume of the solid that is generated when the region enclosed by $y = x^3, y = 1, x = 0$ is revolved about $y = 1$, is

(a) $\frac{9\pi}{14}$

(b) $\frac{7\pi}{15}$

(c) $\frac{15\pi}{21}$

(d) $\frac{3\pi}{14}$

(e) $\frac{17\pi}{14}$

19. The volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the $x-$ axis are squares is

(a) $\frac{1}{30}$

(b) $\frac{1}{12}$

(c) $\frac{1}{18}$

(d) $\frac{1}{36}$

(e) $\frac{1}{24}$

20. Let m and M be the absolute minimum and the absolute maximum values respectively, of an integrable function f over a closed interval $[3, 5]$. If an estimation, based on m and M, of the integral $\int_3^5 f(x)dx$ lies in the interval $[A, B]$, then $A + B =$

(a) $2(M + m)$

(b) $2(M - m)$

(c) $8(M + m)$

(d) $8(M - m)$

(e) $2Mm$