King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
T-131
Saturday 05/10/2013

Net Time Allowed: 120 minutes

MASTER VERSION

1. If f is an integrable function and
$\int_1^3 (2 - f(t))\,dt = \int_3^5 (t + f(t))\,dt$, then $\int_1^5 f(t)\,dt =$

(a) -4

(b) -2

(c) -6

(d) 6

(e) 8

2. If g is a continuous function such that
$\int_0^{2x} e^{t/2} g(t)dt = x\,e^x$, then $g(4) =$

(a) $\frac{3}{2}$

(b) $\frac{5}{2}$

(c) 3

(d) 5

(e) $\frac{7}{2}$

3. The area of the surface obtained by rotating the curve $y = \sqrt{x}$, $4 \le x \le 9$ about the x-axis is

(a) $\frac{\pi}{6}(37\sqrt{37} - 17\sqrt{17})$

(b) $\frac{\pi}{6}(37\sqrt{37} - 17)$

(c) $\frac{\pi}{6}(37\sqrt{37} - \sqrt{17})$

(d) $\frac{\pi}{6}(37 - 17\sqrt{17})$

(e) $\frac{\pi}{6}(\sqrt{37} - 17\sqrt{17})$

4. The length of the curve $y = \int_0^x \sqrt{\cos 2t}\, dt$ from $x = 0$ to $x = \frac{\pi}{4}$ is

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

5. If the average value of the function $f(x) = x^2 + 1$ on the interval $[-1, b]$ is equal to 2, then $b =$

(a) 2

(b) 3

(c) 4

(d) 1

(e) 0

6. The area of the surface generated by rotating the curve $x = \sqrt{a^2 - y^2}$, $0 \leq y \leq a/2$, about the y-axis is

(a) πa^2

(b) $\dfrac{\pi a^3}{3}$

(c) πa

(d) $3\pi a$

(e) $2\pi a$.

7. If $y = f(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = \sec 2x \tan 2x$, $y\left(\frac{\pi}{6}\right) = 3$, then $y(0) =$

(a) $\frac{5}{2}$

(b) $\frac{7}{2}$

(c) $\frac{9}{2}$

(d) $\frac{11}{2}$

(e) $\frac{13}{2}$

8. The base of a solid is bounded by the curves $y = x^2, y = 0$ and $x = 1$. If the cross-sections perpendicular to the x-axis are semi-circles, then the volume of the solid is

(a) $\frac{\pi}{40}$

(b) $\frac{\pi}{5}$

(c) $\frac{\pi}{6}$

(d) $\frac{1}{10}$

(e) $\frac{1}{5}$

9. The region bounded by the curve of $y = \sqrt[3]{x}$ and the lines $y = 0$ and $x = 8$ is revolved about the x-axis. The volume of the solid generated is

(a) $\frac{96}{5}\pi$

(b) π

(c) $\frac{92}{3}\pi$

(d) 8π

(e) $\frac{32}{5}\pi$

10. The region bounded by the curve $y = x^3$ and the line $y = 4x$ in the first quadrant is revolved about the line $y = 8$. The volume of the solid generated is given by

(a) $\pi \int_0^2 (x^6 - 16x^3 - 16x^2 + 64x)\, dx$

(b) $\pi \int_0^2 (9 - 16x^2 + 8x^4 - x^6)\, dx$

(c) $\pi \int_0^2 (9 - x^6 - 6x^3 + 16x^2 - 24x)\, dx$

(d) $\pi \int_0^2 (9 + 16x^2 - 8x^4 + x^6)\, dx$

(e) $\pi \int_0^2 (x^6 - 12x^3 - 24x^2 + 16x)\, dx$

11. The area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ is equal to

(a) 18

(b) 16

(c) 14

(d) 12

(e) 10

12. The volume of the solid generated by rotating the region bounded by the curves $y = x^2$ and $x = y^2$ about the line $x = 2$ is given by

(a) $2\pi \int_0^1 (2 - x)(\sqrt{x} - x^2)\, dx$

(b) $2\pi \int_0^1 (2 + x)(x^2 - x)\, dx$

(c) $2\pi \int_0^1 (x - 2)(x^2 - x^4)\, dx$

(d) $2\pi \int_0^1 (x - 2)(x^2 + x^4)\, dx$

(e) $2\pi \int_0^1 (x - x^4)\, dx$

13. By interpreting the integral $\int_{-2}^{2}(|\,x\,|+\sqrt{4-x^2})\,dx$ in terms of areas, its value is equal to

(a) $4+2\pi$

(b) $2+\pi$

(c) $4+\pi$

(d) $3+2\pi$

(e) $3+\pi$

14. $\int 60\,x^7\,\sqrt{x^4+1}\,dx =$

(a) $6(x^4+1)^{5/2}-10(x^4+1)^{3/2}+C$

(b) $5x^8(x^4+1)^{3/2}+C$

(c) $40(x^4+1)^{5/2}-24(x^4+1)^{3/2}+C$

(d) $\frac{15}{2}x^8+24(x^4+1)^{5/2}+C$

(e) $30(x^4+1)^{3/2}-15(x^4+1)^{5/2}+C$

15. $\displaystyle\int \frac{x-1}{\sqrt{1-x^2}}\,dx =$

(a) $-\sqrt{1-x^2} - \sin^{-1} x + C$

(b) $3\sqrt{1-x^2} - \sin^{-1} x + C$

(c) $-2\sqrt{1-x^2} + \sin^{-1} x + C$

(d) $2\sqrt{1-x^2} - \sin^{-1} x + C$

(e) $-(1-x^2) - \sin^{-1} x + C$

16. $\displaystyle\int \frac{e^{2t}\tan(e^t) - 1}{e^t}\,dt =$

(a) $\ln|\sec e^t| + e^{-t} + C$

(b) $\ln|\sec e^t| - e^{-t} + C$

(c) $\ln|\sec e^{-t}| - e^{-t} + C$

(d) $\ln|\sec e^t| + e^t + C$

(e) $\ln|\sec e^{-t}| + e^{-t} + C$

17. $\int_{-1}^{0} (x+2)(x+1)^{99}\, dx =$

(a) $\dfrac{201}{10100}$

(b) $\dfrac{101}{102}$

(c) $\dfrac{102}{10100}$

(d) $\dfrac{301}{101}$

(e) $\dfrac{101}{20100}$

18. $\int_{0}^{\frac{\pi}{2}} \dfrac{\sin(2x)}{1+\sin^2 x} \ln(1+\sin^2 x)\, dx =$

(a) $\dfrac{1}{2}(\ln 2)^2$

(b) $\dfrac{1}{2} \ln 2$

(c) $(\ln 2)^2$

(d) $\dfrac{1}{2}$

(e) $\dfrac{1}{2} + \ln 2$

19. The area of the region R bounded by the graphs $y = x$, $y = \frac{1}{x}$, $3y - 2x + 5 = 0$ and above the line $y = -x$ is given by

(a) $\ln 3 + \frac{5}{3}$

(b) $\ln 2 + \frac{1}{3}$

(c) $\ln 3 + 2$

(d) $\ln 2 + 3$

(e) $\ln 5 + 4$

20. The $\lim\limits_{x \to 0} \left(\frac{1}{x - \sin x} \int_0^x t \sin t \, dt \right)$

(a) is equal to 2

(b) is equal to 0

(c) is equal to π

(d) is equal to $-\pi$

(e) does not exist