(show all your work and circle one letter to get a full mark or you will get zero)

1)

If $y = x^{\ln(x+1)}$ then $y'(1) =$

(a) $1 + 2^{\ln 2}$

(b) $(1 + \ln 2)^{\ln 2}$

(c) $\ln 2$

(d) $2\ln 2$

(e) 0

(f)   none of the above

$$\ln y = \ln(x+1) \cdot \ln x$$

$$\frac{y'}{y} = \frac{1}{x+1}\ln x + \frac{1}{x}\ln(x+1)$$

$$y'(x) = y(x)\left[\frac{1}{x+1}\ln x + \frac{1}{x}\ln(x+1)\right]$$

$$y'(1) = y(1)\left[\frac{1}{2}(0) + (1)\ln 2\right]$$

$$y(1) = (1)^{\ln 2} = 1$$

$$\Rightarrow y'(1) = (1)\left[0 + \ln 2\right] = \ln 2$$

2) If $f(x) = x\cos^{-1}(\sqrt{1-x^2})$ then $f'(\tfrac{1}{2}) =$

(a) $\dfrac{\pi}{3} - \dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{3}} - \dfrac{\pi}{3}$

(c) $\dfrac{1}{\sqrt{3}} - \dfrac{\pi}{6}$

(d) $\dfrac{\pi}{6} - \dfrac{\pi}{\sqrt{3}}$

(e) $\dfrac{\pi}{6} - \dfrac{1}{\sqrt{3}}$

(f) none of the above

$$f'(x) = \cos^{-1}(\sqrt{1-x^2}) + x\left[\cos^{-1}(\sqrt{1-x^2})\right]'$$

$$f'(x) = \cos^{-1}(\sqrt{1-x^2}) + x\left[-\frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\right]\cdot\left[\sqrt{1-x^2}\right]'$$

$$= \cos^{-1}(\sqrt{1-x^2}) + x\left[-\frac{1}{x^2}\right]\left[\frac{-2x}{2\sqrt{1-x^2}}\right]$$

$$= \cos^{-1}(\sqrt{1-x^2}) + \frac{1}{\sqrt{1-x^2}}$$

$$f'(\tfrac{1}{2}) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{\sqrt{3}/2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{2}{\sqrt{3}}$$

$$= \frac{\pi}{6} + \frac{2}{\sqrt{3}}$$

3) If $f(x) = \ln(x^2+4)^{-3} - 3x\cot^{-1}(\frac{x}{2})$ then find $f'(-2) =$

(a) $-\dfrac{1}{2} + \dfrac{3\pi}{4}$

(b) $-\dfrac{3\pi}{2}$

(c) $\dfrac{3\pi}{4} - \dfrac{5}{6}$

(d) $\dfrac{3\pi}{4} - 3$

(e) $\dfrac{3\pi}{4}$

(f) none of the above

$$f(x) = -3\ln(x^2+4) - 3x\cot^{-1}\left(\frac{x}{2}\right)$$

$$f'(x) = -3\,\frac{2x}{x^2+4} - 3\cot^{-1}\left(\frac{x}{2}\right) - 3x\left(\cot^{-1}\frac{x}{2}\right)'$$

$$= \frac{-6x}{x^2+4} - 3\cot^{-1}\left(\frac{x}{2}\right) - 3x\left(\frac{-\frac{1}{2}}{1+\frac{x^2}{4}}\right)$$

$$= \frac{-6x}{x^2+4} - 3\cot^{-1}\left(\frac{x}{2}\right) + \frac{3}{2}\left(\frac{x}{1+\frac{x^2}{4}}\right)$$

$$f'(-2) = \frac{12}{8} - 3\cot^{-1}(-1) + \frac{3}{2}\,\frac{-2}{(1+1)}$$

$$= \frac{3}{2} - 3\left(-\frac{\pi}{4}\right) - \frac{3}{2}$$

$$= +\frac{3\pi}{4}$$