

2) If  $F(x) = xf(g(x^2))$  where

$$g(4) = 2, g'(4) = -3, f(2) = 1, f'(4) = -6, f'(2) = -2$$

then  $F'(2) =$

- (a) -23
- (b) -15
- (c) 13
- (d) 49
- (e) -71
- (f) none of the above

$$\begin{aligned} F'(x) &= f(g(x^2)) + x [f(g(x^2))]' \\ &= f(g(x^2)) + x f'(g(x^2)) \cdot g'(x^2) \cdot 2x \\ &= f(g(x^2)) + 2x^2 f'(g(x^2)) \cdot g'(x^2) \end{aligned}$$

$$\begin{aligned} F'(2) &= f(g(4)) + 8 f'(g(4)) \cdot g'(4) \\ &= f(2) + 8 f'(2) \cdot (-3) \\ &= (1) + 8(-2)(-3) \\ &= 1 + 48 = 49 \end{aligned}$$

3) If  $x^3 - y^3 = 1$  then find  $y''(-\sqrt[3]{7}) =$

- (a)  $-\sqrt[3]{7}/4$
- (b)  $-\sqrt[3]{7}/8$
- (c)  $-\sqrt[3]{7}/16$
- (d)  $-\sqrt[3]{7}/32$
- (e)  $-\sqrt[3]{7}/64$
- (f) none of the above

$$y''(-\sqrt[3]{7}) =$$

$$3x^2 - 3y^2 y' = 0 \Rightarrow y' = \frac{x^2}{y^2}$$

$$y'' = \frac{2xy^2 - 2yy'x^2}{y^4} = \frac{2xy^2 - 2y(\frac{x^2}{y^2})x^2}{y^4}$$

$$= \frac{2xy^3 - 2x^4}{y^5} = \frac{-2x(x^3 - y^3)}{y^5}$$

$$= \frac{-2x}{y^5} \Rightarrow y'' = -2x/y^5$$

$$\begin{aligned} \text{if } x = -\sqrt[3]{7} &\Rightarrow (-\sqrt[3]{7})^3 - y^3 = 1 \\ &\Rightarrow -7 - y^3 = 1 \Rightarrow y^3 = -8 \\ &\Rightarrow y = -2 \end{aligned}$$

$$\begin{aligned} \text{if } (x, y) = (-\sqrt[3]{7}, -2) &\Rightarrow y''(-\sqrt[3]{7}) = \frac{(-2)(-\sqrt[3]{7})}{(-2)^5} = \frac{2\sqrt[3]{7}}{-2^5} = -\frac{\sqrt[3]{7}}{16} \end{aligned}$$

(show all your work and circle one letter to get a full mark or you will get zero)

1)

If  $f(x) = \tan\left(x + \frac{\pi}{2}\right) \csc(2x)$  then  $f'\left(-\frac{\pi}{4}\right) =$

- (a)  $-16\sqrt{2}/(16 + \pi^2)$   
 (b)  $-32\sqrt{2}/(32 + \pi^2)$   
 (c)  $-2\sqrt{2}/(2 + \pi^2)$   
 (d)  $2\sqrt{2}/(2 + \pi^2)$   
 (e)  $16\sqrt{2}/(16 + \pi^2)$   
 (f) none of the above

$$f'(x) = \sec^2\left(x + \frac{\pi}{2}\right) \csc(2x) - 2 \csc(2x) \cot(x) \tan\left(x + \frac{\pi}{2}\right)$$

$$\begin{aligned} f'\left(\frac{\pi}{4}\right) &= \sec^2\left(\frac{\pi}{4}\right) \csc\left(-\frac{\pi}{2}\right) - 2 \csc\left(-\frac{\pi}{2}\right) \cot\left(-\frac{\pi}{2}\right) \tan\left(\frac{\pi}{4}\right) \\ &= (\sqrt{2})^2 (-1) - 2(-1)(0)(1) \\ &= (2)(-1) = -2 \end{aligned}$$