

1) Let $y = g(x) = x^2 - \frac{1}{x}$. Use limits to find the instantaneous rate of change of y with respect to x at $x = 1$.

$$\begin{aligned} \text{Inst. rate of change} &= g'(1) \\ g'(1) &= \lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h} = \lim_{h \rightarrow 0} \frac{(1+h)^2 - \frac{1}{1+h} - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h)^3 - 1}{h(1+h)} = \lim_{h \rightarrow 0} \frac{h^3 + 3h^2 + 3h + 1 - 1}{h(1+h)} \\ &= \lim_{h \rightarrow 0} \frac{h(h^2 + 3h + 3)}{h(1+h)} = \frac{0+0+3}{1+0} = 3 \end{aligned}$$

2) For what values of a , b , and c is $f(x)$ continuous on the closed interval $[2, 4]$.

$$f(x) = \begin{cases} c & \text{if } x=2 \\ ax-b & \text{if } 2 < x < 3 \\ 2 & \text{if } x=3 \\ \frac{a}{3}x+b & \text{if } 3 < x \leq 4 \end{cases}$$

For $f(x)$ to be cont. on $[2, 4]$ we need:

(*) $f(x)$ cont. at $x=3$

(*) $f(x)$ cont. from right at endpoint $x=2$

This means:

$$(1) \lim_{x \rightarrow 3^+} f(x) = 2 \rightarrow a+b=2$$

$$(2) \lim_{x \rightarrow 3^-} f(x) = 2 \rightarrow 3a-b=2$$

$$(3) \lim_{x \rightarrow 2^+} f(x) = c \rightarrow 2a-b=c \Rightarrow c=1$$

