

1) Find all the vertical and horizontal asymptotes

$$f(x) = \frac{\sqrt{16x^2 + 7}}{|x-3|}$$

Domain of  $f(x)$  is  $\mathbb{R} - \{3\}$

$$\begin{aligned} \lim_{x \rightarrow 3^+} f(x) &= \lim_{x \rightarrow 3^+} \frac{\sqrt{16x^2 + 7}}{x-3} \\ &= \left( \lim_{x \rightarrow 3^+} \sqrt{16x^2 + 7} \right) \left( \lim_{x \rightarrow 3^+} \frac{1}{x-3} \right) = (\sqrt{9 \times 16 + 7}) (+\infty) \\ &= +\infty \end{aligned}$$

$\Rightarrow x=3$  is V.A

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{\sqrt{16x^2 + 7}}{|x-3|} &= \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2} \sqrt{16 + \frac{7}{x^2}}}{(x-3)} \quad (\text{since } x > 3) \\ &= \lim_{x \rightarrow +\infty} \frac{x \sqrt{16 + \frac{7}{x^2}}}{x-3} \quad (\text{divid by } x) \\ &= \lim_{x \rightarrow +\infty} \frac{\sqrt{16 + \frac{7}{x^2}}}{1 - \frac{3}{x}} = \frac{\sqrt{16+0}}{1-0} = 4 \end{aligned}$$

$$\begin{aligned} \text{Also, } \lim_{x \rightarrow -\infty} f(x) &= \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{16 + \frac{7}{x^2}}}{(3-x)} \quad (x \rightarrow -\infty) \\ &= \lim_{x \rightarrow -\infty} \frac{-x \sqrt{16 + \frac{7}{x^2}}}{(3-x)} = \lim_{x \rightarrow -\infty} \frac{-\sqrt{16 + \frac{7}{x^2}}}{\frac{3}{x} - 1} \\ &= \frac{-\sqrt{16+0}}{0-1} = \frac{-4}{-1} = 4 \\ \Rightarrow y=4 &\text{ is H.A} \end{aligned}$$

2) Let  $f(x) = \sqrt{19-x}$ . Find  $\delta > 0$  such that if  $0 < |x-10| < \delta$  then  $|f(x)-3| < 1$

$$f(x) = \sqrt{19-x}, \quad L=3, \quad a=10, \quad \varepsilon=1$$

$$\begin{aligned} L + \varepsilon = 4 &\rightarrow \sqrt{19-x} = 4 \Rightarrow 19-x = 16 \\ &\Rightarrow x = 3 \end{aligned}$$

$$\begin{aligned} L - \varepsilon = 2 &\rightarrow \sqrt{19-x} = 2 \Rightarrow 19-x = 4 \\ &\Rightarrow x = 15 \end{aligned}$$

$$\Rightarrow \delta = \min\{7, 5\} = 5$$

