Name: KEY	Quiz-1(class)	Sec: 14, 22	ID:

1. Evaluate the limit if it exists. 8

(a) $\lim_{x\to1}\frac{\sqrt{2}-\sqrt{x^2+1}}{x-1}\times\frac{\sqrt{2}+\sqrt{x^2+1}}{\sqrt{2}+\sqrt{x^2+1}} = \lim_{x\to1}\frac{2-(x^2+1)}{(x-1)(\sqrt{2}+\sqrt{x^2+1})}$

$\lim_{x\to1}\frac{1-x^2}{(x-1)(\sqrt{2}+\sqrt{x^2+1})} = \lim_{x\to1}\frac{-(\cancel{x-1})(x+1)}{(\cancel{x-1})(\sqrt{2}+\sqrt{x^2+1})}$ 8

$= \frac{-2}{2\sqrt{2}} = -1/\sqrt{2}$ 9

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(b) $\lim_{x\to0}\frac{x+x\cos x}{\sin x\cos x} = \lim_{x\to0}\frac{\frac{x}{x}+\frac{x\cos x}{x}}{\frac{\sin x}{x}\cos x} = \lim_{x\to0}\frac{1+\cos x}{\frac{\sin x}{x}\cos x}$

$= \frac{1+\lim_{x\to0}\cos x}{\left(\lim_{x\to0}\frac{\sin x}{x}\right)\left(\lim_{x\to0}\cos x\right)} = \frac{1+1}{(1)(1)} = \frac{2}{1} = 2$ 9

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(c) $\lim_{x\to0}\left(\frac{|x|+1}{[|x|]+[|-x|]}\right)$, where $[|.|]$ denotes the greatest integer function.

$\lim_{x\to0}(|x|+1) = 1$ 10

$\left.\begin{aligned}\lim_{x\to0^+}([\![x]\!]+[\![-x]\!]) &= (0)+(-1) = -1\\ \lim_{x\to0^-}([\![x]\!]+[\![-x]\!]) &= (-1)+(0) = -1\end{aligned}\right\} \Rightarrow \lim_{x\to0} = -1$ 10

Hence $\lim_{x\to0}\left(\frac{|x|+1}{[\![x]\!]+[\![-x]\!]}\right) = \frac{\lim_{x\to0}(|x|+1)}{\lim_{x\to0}([\![x]\!]+[\![-x]\!])} = \frac{1}{-1} = -1$ 10