Name: KEY	Quiz-1(class)	Sec: 14, 22	ID: KEY

1. Evaluate the limit if it exists. 8

(a) $\lim_{x\to 1}\frac{\sqrt{2}-\sqrt{x^2+1}}{1-x} \times \frac{\sqrt{2}+\sqrt{x^2+1}}{\sqrt{2}+\sqrt{x^2+1}} = \lim_{x\to 1}\frac{2-(x^2+1)}{(1-x)(\sqrt{2}+\sqrt{x^2+1})}$

$= \lim_{x\to 1}\frac{1-x^2}{(1-x)(\sqrt{2}+\sqrt{x^2+1})} = \lim_{x\to 1}\frac{\cancel{(1-x)}(1+x)}{\cancel{(1-x)}(\sqrt{2}+\sqrt{x^2+1})} = \frac{2}{\sqrt{2}+\sqrt{2}}$ 8

$= \frac{2}{2\sqrt{2}} = 1/\sqrt{2}$ 9

(b) $\lim_{x\to 0}\frac{x-x\cos x}{\sin^2 x} = \lim_{x\to 0}\frac{x(1-\cos x)}{1-\cos^2 x}$ 8 $= \lim_{x\to 0}\frac{x\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}$ 8

$= \lim_{x\to 0}\frac{x}{1+\cos x} = \frac{0}{1+1} = \frac{0}{2} = 0$ 9

(d) $\lim_{x\to 0}\left(\frac{|2x|+|-2x|-2}{[|x|]+[|-x|]}\right)$, where [|.|] denotes the greatest integer function.

$\lim_{x\to 0}(|2x|+|-2x|-2) = -2$ 10

$\left.\begin{aligned}\lim_{x\to 0^+}([\![x]\!]+[\![-x]\!]) &= (0)+(-1) = -1\\ \lim_{x\to 0^-}([\![x]\!]+[\![-x]\!]) &= (-1)+(0) = -1\end{aligned}\right\} \Rightarrow \lim_{x\to 0}([\![x]\!]+[\![-x]\!]) = -1$ 10

Hence, $\lim_{x\to 0}\left(\frac{|2x|+|-2x|-2}{[\![x]\!]+[\![-x]\!]}\right) = \frac{-2}{-1} = 2$ 10