

(show all your work and circle one letter to get a full mark or you will get zero)

1) which one of the following statements is FALSE about the function $f(x) = (x-2)x^2(x+2)$

- (a) The function f is concave up on the interval $(1,2)$
- (b) The function f is concave down on $(-0.5, 0.5)$
- (c) The function f has no inflection point
- (d) f has only one local maximum
- (e) The function f is concave up on the interval $(-2,-1)$
- (f) The function f has local minimum at $x = \sqrt{2}$
- (g) none of the above

$$f(x) = x^2(x^2 - 4) = x^4 - 4x^2$$

$$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$$

$$= 4x(x - \sqrt{2})(x + \sqrt{2})$$

$$f''(x) = 12x^2 - 8 = 4(3x^2 - 2)$$

$$= 4(\sqrt{3}x - \sqrt{2})(\sqrt{3}x + \sqrt{2})$$

f' signs: $- \vee + \wedge - \vee +$
 f'' signs: $+ | - | +$
 Inflection points: ≈ -0.81 and ≈ 0.81

- (a) True
- (b) True
- (c) False
- (d) True
- (e) False
- (f) ~~False~~ True
- (g) False

2) If the function $f(x) = ax^3 + bx + c$ has a local maximum value of 2 at $x=1$ then which of the following is TRUE for the function (you may select more than one)

- (a) $abc > 0$
- (b) $a + b + c = 0$
- (c) f concave up on $(-10,-5)$
- (d) f is increasing on $(-1,-0.5)$
- (e) $x=0$ is an inflection point.
- (f) none of the above

local max at $x=1 \Rightarrow f'(1) = 0, f''(1) < 0$
 local max value is 2 $\Rightarrow f(1) = 2$

$$f(1) = 2 \Rightarrow a + b + c = 2$$

$$f'(1) = 0 \Rightarrow 3a + b = 0$$

$$f''(1) < 0 \Rightarrow 6a < 0 \Rightarrow a < 0 \text{ (negative)}$$

$$\begin{cases} b = -3a \\ c = 2 + 2a \end{cases}$$

$$f'(x) = 3ax^2 + b$$

$$f''(x) = 6ax$$

f'	\ominus	\oplus	\ominus	f''	\oplus	\ominus
$3a$	$-$	$-$	$+$	$6a$	$-$	$-$
$x-1$	$-$	$+$	$+$	x	$-$	$+$
$x+1$	$-$	$+$	$+$	x	$-$	$+$

Now, $f(x) = ax^3 - 3ax + (2+2a)$

$$f(x) = a(x^3 - 3x + 2) + 2$$

$$= a(x-2)(x-1) + 2$$

$$f'(x) = 3ax^2 - 3a = 3a(x^2 - 1) = 3a(x-1)(x+1)$$

$$f''(x) = 6ax$$

3) If $f(x) = \frac{x^3 + 2x^2 - 1}{(x+1)^2}$, then an equation of the oblique asymptote for the graph of f is

- (a) $y - x = 0$
- (b) $y - x - 1 = 0$
- (c) $y + x = 0$
- (d) $y - x - 1 = 0$
- (e) f does not have an oblique asymptote
- (f) none of the above

$$\frac{x^3 + 2x^2 - 1}{x^2 + 2x + 1} \begin{array}{r} x \\ \hline x^3 + 2x^2 - 1 \\ \underline{+x^3 + 2x^2 + x} \\ -x - 1 \end{array}$$

$$f(x) = x + \frac{-x-1}{(x+1)^2}$$

So $y = x$ is an oblique asymptote