Name:	A	ID:	Sec: 14 22

MATH-101 Term-131 IN-Class-QUIZ-10

(1) If $y = x^2 \sin x$ then $y'(\frac{\pi}{2}) =$

(a) 2π
(b) π^2
(c) π
(d) $-\pi$
(e) $-\pi^2$

$y' = 2x\sin x + x^2 \cos x$

$y'(\frac{\pi}{2}) = 2(\frac{\pi}{2})(1) + 0$

$= \pi$

(2) If $g(x) = \sec^{-1}(2x)$ then $g'(\frac{1}{\sqrt{2}}) =$

(a) $\sqrt{5}$
(b) $1/\sqrt{5}$
(c) 0
(d) $1/\sqrt{2}$
(e) $\sqrt{2}$

$g'(x) = \frac{2}{|2x|\sqrt{4x^2-1}}$

$= \frac{1}{|x|\sqrt{4x^2-1}}$

$g'(\frac{1}{\sqrt{2}}) = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{1}} = \sqrt{2}$

(3) If $g(x) = \ln(x^2 + 2x)$ then $g'(1) =$

(a) 4/3
(b) -4/3
(c) -3/4
(d) 3/4
(e) 0

$g'(x) = \frac{2x+2}{x^2+2x}$

$g'(1) = \frac{4}{3}$

(4) If $g(x) = \begin{cases} bx + a & x > -1 \\ ax^2 - 1 & x \le -1 \end{cases}$

is differentiable everywhere, then

$4a + b =$

(a) 1
(b) 2
(c) -1
(d) -2
(e) 0

$g'(x) = \begin{cases} b & x > -1 \\ 2ax & x < -1 \end{cases}$

g cont $\Rightarrow -b + a = a - 1$

g diff $\Rightarrow b = -2a$

$\Rightarrow b = 1, a = -\frac{1}{2}$

(5) If at time t, the position of a body moving along the s-axis is $s(t) = \frac{1}{3}t^3 - 4t$, then the total distance travelled by the body from t = 0 to t = 3

(a) 12
(b) 24
(c) 25/3
(d) 23/3
(e) 48

$v(t) = t^2 - 4 = (t-2)(t+2)$

0 2 3

$s(0) = 0$, $s(2) = \frac{8}{3} - 8$

$s(3) = 9 - 12 = -3$

total dist $= |\frac{8}{3} - 8| + |5 - \frac{8}{3}|$

$= 8 - \frac{8}{3} + 5 - \frac{8}{3} = 13 - \frac{16}{3} = \frac{23}{3}$

(6) If $y = \frac{4x-2}{3x+1}$, then $y'''(0) =$

(a) 540
(b) -300
(c) -540
(d) 300
(e) -180

$y' = \frac{4(3x+1) - 3(4x-2)}{(3x+1)^2}$

$= \frac{10}{(3x+1)^2}$

$y'' = (10)(-2)(3x+1)^{-3}(3)$

$= -60(3x+1)^{-3}$

$y''' = 180(3x+1)^{-4}(3)$

$y'''(0) = (180)(3) = 540$

(7) The volume of a cube is increasing at the rate of 270 cm3/min at the instant its edges are 3cm long. At the same instant, the rate at which the lengths of the edges is changing is equal to

(a) 6 cm/min
(b) 7 cm/min
(c) 8 cm/min
(d) 9 cm/min
(e) 10 cm/min

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$270 = 3(3)^2 \frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{270}{27} = 10$

(10) If $y = x^{x-\ln x}$, then $\frac{y'(1)}{y(1)} =$

(a) 2
(b) 1
(c) -2
(d) -1
(e) 0

$\ln y = (x - \ln x)\ln x$

$\frac{y'}{y} = (1 - \frac{1}{x})\ln x + (x - \ln x)\frac{1}{x}$

$\frac{y'(1)}{y(1)} = 0 + (1-0)(1) = 1$

(11) The slope of the normal line to the curve $2x^2 \sin^2 y + 3\sqrt{2}\cos y = 4$

$4x\sin^2 y + 4x^2 \sin y \cos y \cdot y' - 3\sqrt{2}\sin y \cdot y'$

at the point $(1, \pi/4)$ is

(a) -1/2
(b) 1/2
(c) -1/3
(d) -1/4
(e) 2/3

$y' = \frac{-4x\sin^2 y}{\sin y(4x^2\cos y - 3\sqrt{2})}$

$y' = \frac{-(4)(1)(\frac{1}{2})}{(\frac{1}{\sqrt{2}})(4(\frac{1}{\sqrt{2}}) - 3\sqrt{2})}$

$y'(1) = \frac{-2}{2-3} = 2$

normal slope $= -\frac{1}{2}$

For #8 and #9, SHOW all your work and CIRCLE the letter to get a full mark or you will get zero

8 By using linearization we approximate $\sqrt[3]{\frac{7}{27}} \approx$

(a) 21/36
(b) 22/36
(c) 23/36
(d) 24/36
(e) 25/36

$f(x) = \sqrt[3]{x}$, $a = 8/27$

$f'(x) = \frac{1}{3}\frac{1}{\sqrt[3]{x^2}}$, $f(a) = 2/3$, $f'(a) = \frac{1}{3}\frac{1}{(2/3)^2} = \frac{3}{4}$

$L(x) = 2/3 + 3/4(x - 8/27)$

$\sqrt[3]{\frac{7}{27}} = f(\frac{7}{27}) \approx L(\frac{7}{27}) = \frac{2}{3} + \frac{3}{4}(-\frac{1}{27})$

$\frac{2}{3} - \frac{1}{36} = \frac{24-1}{36} = \frac{23}{36}$

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If $L(x) = ax + b$ is a linearization of the function $f(x) = \tan^{-1} x$ at $x = 1$ then $4a + 4b =$

(a) -2π
(b) $-\pi$
(c) 2π
(d) π
(e) 0

$f'(x) = \frac{1}{1+x^2}$, $f(1) = \tan^{-1}(1) = \frac{\pi}{4}$

$f'(1) = \frac{1}{2}$

$L(x) = \frac{\pi}{4} + \frac{1}{2}(x-1)$

$= \frac{1}{2}x + (\frac{\pi}{4} - \frac{1}{2})$

$a = \frac{1}{2}$, $b = \frac{\pi}{4} - \frac{1}{2} \Rightarrow 4a + 4b = \pi$