King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

CODE 001 **Math 101** **CODE 001**

Final Exam

093

Tuesday, August 24, 2010

Net Time Allowed: 180 minutes

Name: ____

ID: ______________________ Sec: ______________________.

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The slope of the tangent line to the curve $y = x^3 - x + 1$ at $x = 1$ is

(a) $\lim\limits_{x\to 1} \dfrac{x^3 - x - 2}{x - 1}$

(b) $\lim\limits_{x\to 1} \dfrac{x^3 - x}{x}$

(c) $\lim\limits_{x\to 1} \dfrac{x^3 - x}{x - 1}$

(d) $\lim\limits_{x\to 1} \dfrac{x^3 - x + 2}{x - 1}$

(e) $\lim\limits_{x\to 1} \dfrac{x^3 - x - 1}{x - 1}$

2. Consider $f(x) = \left(e^{\sin x}\right)^{1/x}$. To make the function $f(x)$ continuous at 0, we need to define $f(0)$ to be

(a) 1

(b) ∞

(c) $\dfrac{1}{e}$

(d) e

(e) 0

3. $\lim\limits_{x \to 2^-} \dfrac{|x-2|}{x^2-x-2} =$

(a) $-\dfrac{2}{3}$

(b) does not exist

(c) $\dfrac{2}{3}$

(d) $\dfrac{1}{3}$

(e) $-\dfrac{1}{3}$

4. The functin $f(x) = 3x^2 - x^3 - 1$

(a) has no zeros in $(-1, 1)$

(b) has one complex zero in $(-1, 1)$

(c) has at least two zeros in $(-1, 1)$

(d) has exactly one zero in $(-1, 1)$

(e) has two complex zeros in $(-1, 1)$

5. Consider the function $f(x) = [\![x]\!] \ln x$, where $[\![x]\!]$ is the greatest integer less than or equal to x. Which of the following is **TRUE**?

(a) $\lim_{x\to 1} f(x) = 1$

(b) $\lim_{x\to 1} f(x) = \infty$

(c) $\lim_{x\to 1} f(x)$ does not exist

(d) $\lim_{x\to 1} f(x) = e$

(e) $\lim_{x\to 1} f(x) = 0$

6. The curve $f(x) = e^{x^2+x}$ has

(a) one vertictal tangent line

(b) one horizontal tangent line

(c) no horizontal tangent lines

(d) one horizontal and one vertical tangent lines

(e) two horizontal tangent lines

7. Using the graph of $y = \sin x$, the maximum value of δ such that $\left|\sin x - \frac{1}{2}\right| < \frac{1}{2}$ whenever $\left|x - \frac{\pi}{6}\right| < \delta$ is equal to

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{5\pi}{6}$

(d) $\frac{\pi}{3}$

(e) $\frac{\pi}{6}$

8. If $f(1) = 1$ and $f'(1) = 3$, then $\left.\frac{d}{dx}\left(\frac{f(x)}{x^2}\right)\right|_{x=1}$ is equal to

(a) 1

(b) 2

(c) -1

(d) 0

(e) -2

9. Using differentials to approximate $\tanh(\ln 1.1)$, we get

(a) 1

(b) e

(c) 0.1

(d) -1.1

(e) 1.1

10. The function $f(x) = x^3+3x^2+3x+1$ attains on the interval $[0, 1)$

(a) only one critical number

(b) no absolute maximum and no absolute minimum

(c) an absolute maximum and no absolute minimum

(d) an absolute minimum and no absolute maximum

(e) an absolute maximum and an absolute minimum

11. Using the linearization of $f(x) = \sinh(x-1)$ at $a = 1$ to approximate $\sinh(0.1)$, we get

(a) 0.9

(b) 0.1

(c) -1

(d) -0.1

(e) 1

12. If the circumference of a circle is increasing at a rate of 0.1 cm/min, then the rate at which the area is increasing when the radius is 10 cm equals

(a) 0.1 cm/min

(b) π cm/min

(c) 2π cm/min

(d) 1 cm/min

(e) 0.2π cm/min

13. Suppose that $F(x) = f(g(x))$ and $g(3) = 6,\ g'(3) = 4,\ f'(3) = 2$ and $f'(6) = 7$. Then $F'(3) =$

(a) 12

(b) 8

(c) 28

(d) 21

(e) 6

14. If $y = \tan(2x+1)$ then $y'' - 4yy' =$

(a) $2\sec^2(2x+1)\tan(2x+1)$

(b) $2\sec(2x+1)\tan(2x+1)$

(c) $4\sec(2x+1)\tan(2x+1)$

(d) $4\sec^2(2x+1)\tan(2x+1)$

(e) 0

15. If $f(x) = \log x^2$, then $f^{(101)}(1) =$

(a) $\dfrac{2^{100}}{\log 2}(100!)$

(b) $\dfrac{2}{\ln 10}(100!)$

(c) $-\dfrac{2}{\ln 10}(100!)$

(d) $\dfrac{2}{\ln 10}(101!)$

(e) $2\ln 10(100!)$

16. Using Newton's method to approximate $\sqrt{3}$, and taking $x_1 = 1$ as the first approximation, the third approximation x_3 is

(a) $\dfrac{5}{4}$

(b) 2

(c) -2

(d) $\dfrac{3}{4}$

(e) $\dfrac{7}{4}$

17. $\lim_{x\to 0} \dfrac{x \sin x}{\cos x - 1} =$

(a) 2

(b) -2

(c) does not exist

(d) 0

(e) 1

18. If $y = \coth^{-1}(\sin x^2)$, then $y' =$

(a) $-2x \text{ csch } x^2$

(b) $2x \text{ sech } x^2$

(c) $2x \sec x^2$

(d) $2x \csc x^2$

(e) $-2x \sec x^2$

19. If the point (a, b) on the line $y = 2x + 2$ is closest to the origin, then $5(a + b) =$

(a) 3

(b) -2

(c) -3

(d) 2

(e) 1

20. The equation of the normal line to the curve $xy^2 + x^2y = 2$ at $(1, 1)$ is

(a) $y = x$

(b) $y = x + 1$

(c) $y = 2x - 1$

(d) $y = \frac{1}{2}x + \frac{1}{2}$

(e) $y = -x + 2$

21. If $f''(x) = 2e^x + 3\sin x$, $f(0) = 0$ and $f'(0) = 0$, then $f(\pi) =$

(a) $2e^{\pi} + \pi + 2$

(b) $2e^{\pi} - \pi + 2$

(c) $2e^{\pi} - \pi - 2$

(d) $2e^{\pi} + \pi - 2$

(e) $2e^{\pi}$

22. If $y = (2 + \tan x)^x$, then $y'(0) =$

(a) $\ln 2$

(b) $\frac{1}{2} + \ln 2$

(c) $2\ln 2$

(d) 4

(e) 2

23. Consider the function $f(x) = \dfrac{x-1}{x+1}$. The value of c which satisfies the conclusion of the Mean Value Theorem on the interval $[0, 2]$ is

(a) $1 - \sqrt{3}$

(b) 1

(c) $\sqrt{3}$

(d) $-1 + \sqrt{3}$

(e) $1 + \sqrt{3}$

24. A can company wants to make cylindrical cans (with top) that holds $2000\,\pi\ \text{cm}^3$ of soup. The dimensions of the can which requires the least amount of metal is
(Hint: $V = \pi r^2 h$)

(a) radius $\sqrt{20}$ cm and height 10 cm

(b) radius 10 cm and height 20 cm

(c) radius 10 cm and height 10 cm

(d) radius 20 cm and height 5 cm

(e) radius 20 cm and height 10 cm

25. $\lim_{x \to 0} (1 + \sin x)^{1/x} =$

(a) 1

(b) 0

(c) e

(d) -1

(e) $\frac{1}{e}$

26. The function $f(x) = \dfrac{x-1}{x^2-1}$

(a) has two critical numbers

(b) is decreasing on $(-\infty, -1)$

(c) has two vertical and one horizontal asymptotes

(d) concave up on $(-\infty, 1)$

(e) has one inflection point

27. The graph of $f(x) = \dfrac{x+1}{e^x}$

(a) is concave up on $(0, \infty)$

(b) is concave down on $(1, \infty)$

(c) has no inflection points

(d) is concave up on $(1, \infty)$

(e) is concave down on $(0, \infty)$

28. If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$, then

(a) $F(x) + G(x)$ is an antiderivative of $f(x)$

(b) $F(x) \cdot G(x)$ is an antiderivative of $f(x)$

(c) $\dfrac{F(x)}{G(x)}$ is an antiderivative of $f(x)$

(d) $F(x) - G(x)$ is an antiderivative of $f(x)$

(e) $2F(x) - G(x)$ is an antiderivative of $f(x)$