King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 101 -Major Exam II

Semester 093

Tuesday, August 24, 2010

Net Time Allowed: 120 minutes CODE 001

Name:_____

ID: _____ Sec: _____

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. $\left[\cosh(\ln 2) + \sinh(\ln 2)\right]^2 =$

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\ln 2$

(d) 4

(e) $\ln 4$

2. The equation of the tangent line to the curve $y = \ln x$ which passes through the origin, is

(a) $y = \frac{x}{e} + 1$

(b) $y = ex$

(c) $y = \frac{x}{e}$

(d) $y = ex + 1$

(e) $y = x$

3. If $u = e^{y^3+y}$ and $x = u^3 + u$, then $\left.\dfrac{dy}{dx}\right|_{x=2}$ is

(a) e^2

(b) 2

(c) $\dfrac{1}{4}$

(d) $\dfrac{7}{3}$

(e) $e^2 - 1$

4. $\lim\limits_{x\to 0} \dfrac{\sin(\sin 5x)}{3x} =$

(a) $\dfrac{1}{15}$

(b) $\dfrac{3}{5}$

(c) 3

(d) 5

(e) $\dfrac{5}{3}$

5. $\lim_{x\to 2} \dfrac{2^x + x^2 - 8}{x - 2} =$

(a) $4 \ln 2$

(b) $2^4 \ln 2$

(c) $4 + 2 \ln 2$

(d) $4 + \ln 2$

(e) $4 + 4 \ln 2$

6. If $g(x) = xe^x + \dfrac{x+1}{2 + \ln(x+1)}$, then $g'(0) =$

(a) $\dfrac{5}{2}$

(b) $\dfrac{3}{2}$

(c) $\dfrac{3}{4}$

(d) $\dfrac{5}{4}$

(e) $e + \dfrac{1}{2}$

7. If $y = \coth^{-1}(\cosh x)$, then $y' =$

(a) $\operatorname{sech} x \tanh x$

(b) $-\operatorname{csch} x$

(c) $-\operatorname{sech} x$

(d) $\operatorname{csch} x$

(e) $\operatorname{sech} x$

8. At 12:00 noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.?

(a) $\dfrac{285}{\sqrt{101}}$ km/h

(b) 215 km/h

(c) $10\sqrt{101}$ km/h

(d) $\dfrac{215}{\sqrt{101}}$ km/h

(e) $\dfrac{2150}{\sqrt{101}}$ km/h

9. If $f(x) = x^{\pi} - e^{x} + \pi^{2}$, then $f'(0) =$

(a) 2π

(b) $-1 + 2\pi$

(c) $1 + 2\pi$

(d) -1

(e) 1

10. If $y = \dfrac{\cot x}{1 + x \cot x}$, then $y' = \dfrac{g(x)}{(1 + x \cot x)^{2}}$ where $g(x) =$

(a) 1

(b) $-2 \csc^{2} x$

(c) $-(1 + 2 \cot^{2} x)$

(d) $2 \csc^{2} x$

(e) $1 + 2 \cot^{2} x$

11. The slope of the tangent line to the curve

$$\sin\left(\frac{\pi y}{x}\right) = x - y$$

at the point $(1, 1)$ is

(a) 0

(b) -1

(c) $\dfrac{\pi}{1+\pi}$

(d) 1

(e) $\dfrac{-\pi}{1+\pi}$

12. $\lim\limits_{x\to 0} \dfrac{\tan 7x}{\sin 6x} =$

(a) $\dfrac{7}{3}$

(b) 1

(c) $\dfrac{7}{6}$

(d) $\dfrac{6}{7}$

(e) $\dfrac{3}{7}$

13. Water is being pumped out at a rate of 0.1 m^3/min from a container, taking the shape of an inverted right circular cone with radius 3 m and height 6 m. The rate at which the water level in the container is dropping when the water is 2 m high is

$(V = \frac{1}{3}\pi r^2 h)$

(a) $\frac{1}{5\pi}$ m/min

(b) $\frac{1}{6\pi}$ m/min

(c) $\frac{1}{10\pi}$ m/min

(d) $\frac{2}{5\pi}$ m/min

(e) $\frac{1}{2\pi}$ m/min

14. A particle moves according to the law of motion $s(t) = t^3 - 12t^2 + 36t, \quad t \geq 0$, where t is measured in seconds and s in meters. The particle is speeding up when

(a) $2 < t < 4$ and $t > 6$

(b) $t > 4$

(c) $0 \leq t < 4$ and $t > 6$

(d) $0 \leq t < 2$ and $t > 6$

(e) $0 \leq t < 2$ and $4 < t < 6$

15. If $y = x \ln x$, then $y^{(100)} =$

(a) $-\dfrac{98!}{x^{98}}$

(b) $\dfrac{98!}{x^{99}}$

(c) $\dfrac{98!}{x^{98}}$

(d) $\dfrac{100!}{x^{100}}$

(e) $-\dfrac{98!}{x^{99}}$

16. Which one of the following is an identity?

(a) $\cosh 2x = 1 - 2\sinh^2 x$

(b) $1 + \tanh^2 x = \operatorname{sech}^2 x$

(c) $\cosh x + \sinh x = e^x$

(d) $\cosh 2x = \cosh^2 x - \sinh^2 x$

(e) $\sinh^2 x + \cosh^2 x = 1$

17. If $y = \sin^2(\cos^3 x^5)$, then $y' =$

(a) $-15x^4(\sin x^5)(\cos^2 x^5)\sin(2\cos^3 x^5)$

(b) $-30x^4(\sin x^5)(\cos^2 x^5)\sin(\cos^3 x^5)$

(c) $30x^4\sin(\cos^2 x^5)$

(d) $30\sin(\cos^2 5x^4)$

(e) $-15x^4(\sin x^4)(\cos^2 x^5)\sin(\cos^3 x^5)$

18. Using linear approximation, $\tan 44^\circ$ is estimated to be

(a) $1 - \dfrac{\pi}{90}$

(b) $1 + \pi$

(c) $1 - \pi$

(d) $1 + \dfrac{\pi}{90}$

(e) 1

19. The slope of the normal line to the curve

$$2xy + \ln xy = 1 + 2e \text{ at } (1, e)$$

is

(a) e

(b) $-e$

(c) 1

(d) $-\dfrac{1}{e}$

(e) $\dfrac{1}{e}$

20. If $xy = \cot(xy)$, then $y'' =$

(a) $\dfrac{y}{x^2}$

(b) $-\dfrac{y}{x^2}$

(c) $\dfrac{2y}{x^2}$

(d) 0

(e) $-\dfrac{y}{x}$