1. A particle moves along the curve $xy + y^2 = 35$. As it reaches the point $(2, 5)$ the $x-$coordinate is increasing at a rate of $2\,cm/sec$. How fast is the y-coordinate changing at that instant?

   a) $-\frac{5}{6}\ cm/sec.$
   b) $-3\ cm/sec.$
   c) $-\frac{7}{6}\ cm/sec.$
   d) $3\,cm/sec.$
   e) $\frac{1}{6}\ cm/sec.$

2. The slope of the tangent line to the graph of $y = (1 + 4x^2)tan^{-1}(2x)$ at $x = \frac{1}{2}$ is

   a) $2 + \pi$
   b) $4 + \pi$
   c) $8 + 2\pi$
   d) $2 - \pi$
   e) $1 + 2\pi$

3. If $y = (\sqrt{x} - \frac{1}{\sqrt{x}})^2$, then $y'' =$

a) $2x^{-3}$
b) $2\sqrt{x^3}$
c) $-3x^{-2}$
d) $-2\sqrt{x^3}$
e) $4x^{-3}$

4. If $y = \frac{tan\, x}{1 + x\, tan\, x}$, then $y' = \frac{g(x)}{(1 + x\, tan\, x)^2}$ where $g(x) =$

a) 1
b) $sec\, x\, tan\, x$
c) $sec\, x$
d) $tan\, x$
e) -2

5. The limit $\lim_{x\to 2} \dfrac{tan\,(x-2)}{4x^2-8x}$

a) is equal to $\dfrac{1}{8}$
b) is ∞
c) is $-\infty$
d) is equal to $\dfrac{1}{4}$
e) is equal to 1

6. If $y = \sqrt{sin\,(csc\,\pi x)}$, then the product $2yy'$ is equal to

a) $-\pi\, cos\,(csc\,\pi x).csc\,\pi x.\, cot\,\pi x$
b) π
c) $-\pi$
d) $2sin\,\pi x.\, cos\,\pi x$
e) $cos\,(csc\,\pi x.cot\,\pi x)$

7. If $f'(x) = \lim_{h\to 0} \dfrac{e^{2x}\left(e^{2h}-1\right)}{3h}$, then a possible expression for the function f is

a) $f(x) = \dfrac{1}{3}e^{2x}$
b) $f(x) = \dfrac{1}{3}\left(e^{x}+1\right)^{2}$
c) $f(x) = 3e^{2x}$
d) $f(x) = 3\left(e^{2x}-1\right)$
e) $f(x) = e^{2x} + \frac{1}{3}$

8. If $(0, \alpha)$ are the coordinates of the y-intercept of the tangent line to the graph of $y = \sin^{-1}(2x)$ at $x = \dfrac{\sqrt{2}}{4}$, then $\alpha =$

a) $\dfrac{\pi}{4} - 1$
b) $\dfrac{\pi}{4} - \sqrt{2}$
c) $\dfrac{\pi}{4} - \dfrac{1}{4}$
d) $\dfrac{\pi}{4} + \sqrt{2}$
e) $\pi\sqrt{2}$

9. If a particle is moving according to a law of motion $S(t) = 3\cos\left(\frac{\pi}{2}t\right)$ where t is measured in seconds and S in meters, then the total distance traveled by the particle during the time interval $[0, 5]$ is

   a) 15 meters
   b) 6 meters
   c) 12 meters
   d) 18 meters
   e) 9 meters

10. If $y = \ln\left[\frac{2}{3}\left(x + \sqrt{x^2 - 1}\right)\right]$, then $\frac{dy}{dx} =$

   a) $\frac{1}{\sqrt{x^2 - 1}}$
   b) $\frac{2}{3\sqrt{x^2 - 1}}$
   c) $\frac{3}{2\sqrt{x^2 - 1}}$
   d) $\frac{2}{3}\left(1 + x\sqrt{x^2 - 1}\right)$
   e) $\frac{1}{x + \sqrt{x^2 - 1}}$

11. The volume of a sphere is changing at the rate of $4\pi\, cm^3/sec.$ How fast is the diameter of the sphere changing when its volume is $\frac{32\pi}{3}\, cm^3$?

    [Volume of a sphere $= \frac{4\pi}{3}(radius)^3$].

    a) $\frac{1}{2}\ cm/sec.$
    b) $\frac{3}{2}\ cm/sec.$
    c) $\frac{3}{8}\ cm/sec.$
    d) $\frac{3}{4}\ cm/sec.$
    e) $\frac{9}{8}\ cm/sec.$

12. The slope of the normal line to the curve $e^{\frac{y}{x}} = x - y$ at the point $(1, 0)$ is

    a) -2
    b) 1
    c) -1
    d) $\frac{1}{3}$
    e) -3

13. If $f(x) = x^{\ln x}$, then $f'(e^2) =$

a) $4e^2$
b) $2e^2$
c) e^2
d) $\frac{1}{2}e^2$
e) $\frac{1}{4}e^2$

14. If $f(x) = 2^x \cdot x^2$, then $f'(2) =$

a) $16(1 + \ln 2)$
b) $8(2 + \ln 2)$
c) $16(\ln 2 - 1)$
d) $\ln 16 + ln2$
e) $4(1 + \ln 2)$

15. $\lim_{x\to 0} \dfrac{5x^2}{2x - 2x\cos x + 2\sin^2 3x}$

a) is equal to $\dfrac{5}{18}$

b) does not exist

c) is equal to 0

d) is equal to $\dfrac{5}{3}$

e) is equal to 5

16. If $f(x) = xe^{-x}$, then $f^{(100)}(x) = (Ax + B)e^{-x}$, where A and B are constants and $A + B =$

a) -99

b) 99

c) -2

d) 101

e) -100

17. If the tangent line to the graph of $y = x^2 - 4x + 19$ at the point $P(\alpha, \beta)$, where $\alpha < 0$, passes through the point $(3, 0)$, then $\beta =$

a) 24
b) 18
c) 30
d) 28
e) 36

18. Which one of the following statements is TRUE about the function $f(x) = \sqrt[3]{x}$?

a) The graph of f has vertical tangent at $x = 0$
b) $\lim_{x \to 0^-} f'(x) = -\infty$
c) The graph of f has a vertical asymptote at $x = 0$
d) f is differentiable at $x = 0$
e) The graph of f' lies below the $x-$axis on $(-\infty, 0)$

19. Given the function $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ |x+1| & \text{if } -2 \leq x \leq 8. \\ (x-6)^2 & \text{if } x > 8 \end{cases}$

The sum of all values of x for which the function f is not differentiable is

a) 5

b) 6

c) 0

d) 3

e) 4

20. Which one of the following statements is TRUE?

a) If f' is differentiable at a, then $f''(a)$ exists

(b) If $\lim_{x \to a} f'(x)$ exists, then f' is continuous at a

(c) If f is continuous at a, then f is differentiable at a

(d) If $f(a) = 0$ and $f'(a) = 0$, then $f''(a) = 0$.

(e) If $f'(a)$ does not exist, then f is discontinuous at a.