King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

**Math 101**
**Final Exam**
**Term 122**
**Monday 20/15/2013**
**Net Time Allowed: 180 minutes**

# MASTER VERSION

1. An equation of the tangent line to the curve $y = \dfrac{1}{x^3}$ when $x = -1$ is given by

(a) $y = -3x - 4$

(b) $y = -2x - 3$

(c) $y = -\dfrac{1}{3}x - \dfrac{4}{3}$

(d) $y = \dfrac{1}{3}x + 1$

(e) $y = -3x - 2$

2. Let $\mu$ be a real number not equal to $-1$. Then $\lim\limits_{x \to 1} \dfrac{x^{\frac{\mu}{\mu+1}} - 1}{x - 1} =$

(a) $\dfrac{\mu}{\mu + 1}$

(b) $\dfrac{\mu + 1}{\mu}$

(c) $\dfrac{1}{\mu + 1}$

(d) $\mu + 1$

(e) does not exist

3. $\sum_{k=1}^{7} \left(2k + 6k^2\right) =$

(a) (16)(7)(8)

(b) (15)(7)(8)

(c) (16)(7)

(d) (7)(8)(10)

(e) (7)(8)(14)

4. Using four rectangles and the **midpoint rule**, the area under the graph of $f(x) = 1 + x^2$ from $x = 0$ to $x = 4$ is approximately equal to

(a) 25

(b) 20

(c) 30

(d) 27

(e) 18

5. Suppose that $g(x) \leq f(x) \leq h(x)$ for all $x \neq 2$ and suppose that

$$\lim_{x\to 2} g(x) = \lim_{x\to 2} h(x) = -5.$$

Which one of the following statements is **TRUE**:

(a) $\lim_{x\to 2} f(x) \neq 0$

(b) $f(2)$ must be equal to $-5$

(c) $f(2)$ cannot be equal to $0$

(d) $\lim_{x\to 2} f(x)$ can be different from $-5$

(e) $f(2)$ and $\lim_{x\to 2} f(x)$ must be equal

6. If $f(x) = \dfrac{x^3 + 2x^2 - 1}{(x+1)^2}$, then an equation of the oblique asymptote for the graph of $f$ is

(a) $y - x = 0$

(b) $y - x - 1 = 0$

(c) $y + x = 0$

(d) $y + x + 1 = 0$

(e) $f$ does not have an oblique asymptote

7. If $f(x) = \cot^{-1}\left(\dfrac{1}{x}\right) - \tan^{-1} x$, then $f'(1) =$

(a) $f'(2)$

(b) does not exist

(c) $-1$

(d) $\dfrac{-2}{3}$

(e) $f'(0)$

8. If $y = (\sec x + \tan x)(\sec x - \tan x)$, then $\dfrac{dy}{dx} =$

(a) $0$

(b) $\sec^2 x$

(c) $\sec^3 x$

(d) $1$

(e) $\sec^2 x \, \tan x$

9. The slope of the **normal line** to the curve $2y + \pi\sin(xy) = 2\pi$ at the point $(1, \pi)$ is

(a) $\frac{\pi - 2}{\pi^2}$

(b) $0$

(c) $\frac{1}{\pi}$

(d) $-\frac{2}{\pi^2}$

(e) None of them

10. Newton's method is used to estimate the $x$-coordinate of the point of intersection of the curves $y = \sqrt{x}$ and $y = 1 - x^2$. If we start with $x_0 = 1$, then $x_1 =$

(a) $\frac{3}{5}$

(b) $0$

(c) $\frac{-1}{2}$

(d) $\frac{1}{2}$

(e) $\frac{8}{5}$

11. If the function

$$f(x) = \begin{cases} \dfrac{x+b}{b+1} & x < 0 \\ x^2 + b & x \geq 0 \end{cases}$$

is continuous everywhere, then $f(-1) =$

(a) $-1$

(b) $0$

(c) $2$

(d) $4$

(e) $-3$

12. Let $f(x) = \sqrt{1-3x}$. The **greatest possible** value of $\delta > 0$ for which $\lim\limits_{x \to -1} f(x) = 2$, when $\varepsilon = \dfrac{1}{2}$ is

(a) $\dfrac{7}{12}$

(b) $\dfrac{9}{12}$

(c) $\dfrac{5}{12}$

(d) $\dfrac{5}{2}$

(e) $\dfrac{3}{2}$

13. Evaluate the limit $\lim_{x \to -\infty} (x + \sqrt{x^2 + 2x})$

(a) $-1$

(b) $1$

(c) $-2$

(d) $2$

(e) $0$

14. If $y = (2x)^{\sin(2x)}$, $\dfrac{dy}{dx} =$

(a) $y \left[ \dfrac{\sin(2x)}{x} + 2\ln(2x)\cos(2x) \right]$

(b) $y \left[ \dfrac{\sin(2x)}{2x} + 2\ln(2x)\cos(2x) \right]$

(c) $y \left[ \dfrac{\sin(2x)}{x} - 2\ln(2x)\cos(2x) \right]$

(d) $\sin(2x)\,(2x)^{\sin(2x)-1}\ln(2x)$

(e) $y \left[ \dfrac{\cos(2x)}{x} - 2\ln(2x)\sin(2x) \right]$

15. Evaluate the limit $\lim_{x\to -1^+}\left(\sqrt{x+1}\,\ln(x+1)\right)$

(a) $0$

(b) $1$

(c) $-1$

(d) $\infty$

(e) $-\infty$

16. Let $f(x) = \cos^2 x + \sin x,\ 0 < x < \pi$. Which one of the following statements is **TRUE**:

(a) $f$ is decreasing on the intervals $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ and $\left(\frac{5\pi}{6}, \pi\right)$

(b) $f$ is decreasing on the interval $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(c) $f$ is increasing on the interval $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

(d) $f$ is increasing on the intervals $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(e) $f$ is decreasing on the intervals $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ and $\left(\frac{2\pi}{3}, \pi\right)$

17. If $F(x) = f\left(xf(xf(x))\right)$ where $f(1) = 2, f(2) = 3, f'(1) = 4, f'(2) = 5$ and $f'(3) = 6$, then $F'(1) =$

(a) 198

(b) 0

(c) $-1$

(d) $-200$

(e) 144

18. Let $f(x) = \dfrac{2x^2}{x^2-1}$. Which one of the following statements is **TRUE**:

(a) The graph of $f$ is concave up on $(-\infty, -1) \cup (1, \infty)$

(b) The graph of $f$ is concave up on $(-\infty, -1) \cup (0, 1)$

(c) The graph of $f$ is concave down on $(-1, 1) \cup (1, \infty)$

(d) The graph of $f$ is concave down on $(-1, 0) \cup (1, \infty)$

(e) $f$ has two inflection points

19. Let $f(x) = x^2\sqrt{5 - \frac{x}{2}}, x \in [-2, 2]$. Which one of the following statements is **TRUE**?

(a) $f$ has a local minimum at $x = 0$

(b) $f$ has a local maximum at $x = 8$

(c) the absolute maximum value of $f$ is $8$

(d) $f$ has absolute minimum at $x = -2$

(e) $f$ has no absolute minimum

20. The most general antiderivative of $f(t) = \dfrac{te^{2t} + \sqrt[3]{t}}{t}$ is

(a) $\frac{1}{2}e^{2t} + 3t^{1/3} + C$

(b) $e^{2t} + 3t^{2/3} + C$

(c) $te^{2t} + t^{-2/3} + C$

(d) $2e^{2t} - \frac{3}{2}t^{-5/3} + C$

(e) $\frac{1}{2}e^{2t} - \frac{3}{2}t^{-5/3} + C$

21. The **number** of the critical points of $f(x) = |x^3 - 4x|$ is

(a) 5

(b) 4

(c) 3

(d) 2

(e) 1

22. The real values of $x_0$ and $L$ that satisfy the following limit

$$\lim_{x \to x_0} \frac{\ln(x+1)}{x - x_0} = 3L$$

are

(a) $x_0 = 0;\ L = \frac{1}{3}$

(b) $x_0 = -1;\ L = \frac{1}{2}$

(c) $x_0 = -2;\ L = 1$

(d) $x_0 = 1;\ L = \frac{1}{3}$

(e) $x_0 = \frac{1}{3};\ L = \frac{1}{4}$

23. Approximating $\tan^{-1}(1.01)$ using a linearization of $f(x) = \tan^{-1}(x)$ at a suitably chosen integer near 1.01 is equal to

(a) $\frac{\pi}{4} + 0.005$

(b) $\frac{\pi}{2} + 0.005$

(c) $\frac{\pi}{4} + 0.01$

(d) $\frac{\pi}{2} + 0.01$

(e) $0.005$

24. The value(s) of $c$ satisfying the conclusion of the Mean Value Theorem for the function $f(x) = x + \frac{1}{x}$, on the interval $\left[\frac{1}{2}, 2\right]$ is (are)

(a) $1$

(b) $-1$ and $1$

(c) $\frac{1}{2}$ and $\frac{3}{2}$

(d) $\frac{1}{4}$ and $1$

(e) $1$ and $\frac{7}{4}$

25. Let $a > 0$ and let $f(x) = \frac{x^2}{3} + \frac{x}{a}, a \leq x \leq 2a$. The value of $a$ such that the **average rate of change** of the function $f$ on the interval $[a, 2a]$ is the **smallest possible** is

(a) 1

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) 2

(e) $\sqrt{3}$

26. A surveyor, standing $50ft$ from the base of a building, measures the angle of elevation to the top of the building to be $45°$. How accuratley must the angle be measured for the percentage error in estimating the height of the building to be less than 3%?

(a) 1.5%

(b) 1%

(c) 2%

(d) 2.5%

(e) 3%

27. The volume of a cube is increasing at the rate of 270 $cm^3/min$ at the instant its edges are 6 $cm$ long. At the same instant, the rate at which the lengths of the edges is changing is equal to

(a) $2.5\,cm/min$

(b) $3\,cm/min$

(c) $3.5\,cm/min$

(d) $2\,cm/min$

(e) $4\,cm/min$

28. The least amount of material needed to construct an open-top right circular can that will hold a volume of 1000 $cm^3$ is equal to

(a) $300\pi^{1/3}\,cm^2$

(b) $100\pi^{-2/3}\,cm^2$

(c) $10\pi^{-1/3}\,cm^2$

(d) $400\pi^{1/3}\,cm^2$

(e) $300\pi^{-2/3}\,cm^2$

| Q | MM | V1 | V2 | V3 | V4 |
|---|---|---|---|---|---|
| 1 | a | a | a | e | a |
| 2 | a | d | c | d | c |
| 3 | a | e | e | e | d |
| 4 | a | a | d | c | b |
| 5 | a | a | d | c | b |
| 6 | a | d | a | d | b |
| 7 | a | a | e | a | c |
| 8 | a | a | e | b | e |
| 9 | a | c | a | d | b |
| 10 | a | b | a | c | d |
| 11 | a | a | c | c | c |
| 12 | a | c | e | a | d |
| 13 | a | d | b | c | e |
| 14 | a | a | c | a | d |
| 15 | a | c | c | a | b |
| 16 | a | b | b | c | b |
| 17 | a | a | e | c | d |
| 18 | a | c | c | c | d |
| 19 | a | a | d | c | a |
| 20 | a | d | d | c | a |
| 21 | a | d | e | b | a |
| 22 | a | b | e | a | e |
| 23 | a | c | d | a | c |
| 24 | a | b | e | d | c |
| 25 | a | b | a | b | c |
| 26 | a | a | a | c | a |
| 27 | a | c | b | b | b |
| 28 | a | a | b | e | e |