King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MASTER

MATH 101 - Term 122 - Exam II

MASTER

Duration: 120 minutes

Name: KEY

ID Number: ______________ Section Number: ______________

Check that this exam has 20 questions.

Instructions:

1. Any type of calculators, pagers, or mobile phones are **NOT** allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use erasers attached to the pencil.
4. Write your name, ID number, and section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and section number, make sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

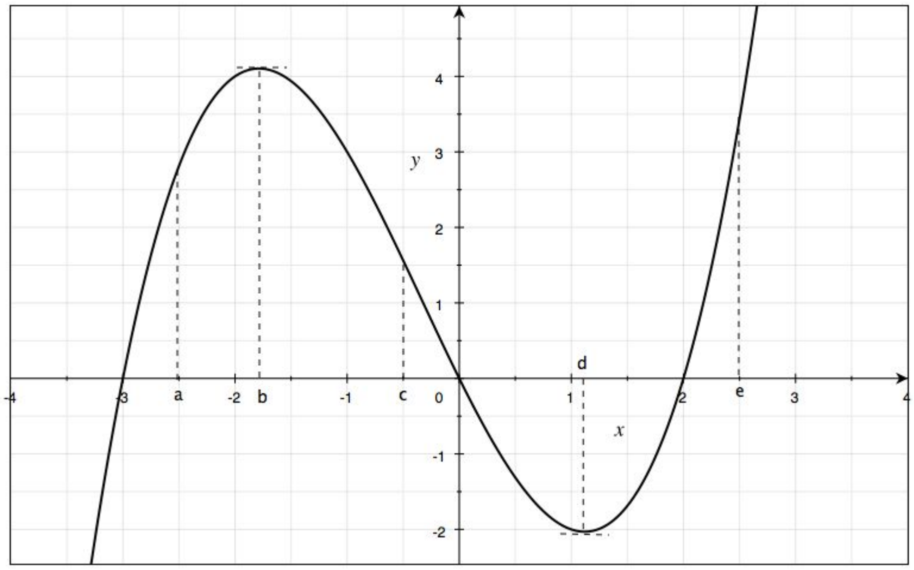
1. Let $f(x) = \dfrac{x+2}{2x+5}$ on the interval $(-\infty, \dfrac{-5}{2})$ and $P(c, f(c))$ be a point on the graph of f. If the tangent line at P is perpendicular to the line $x + y + 1 = 0$, then the y-intercept of the tangent line is

   a) 4
   b) -4
   c) 0
   d) 1
   e) -1

2. If $f(t) = \dfrac{1}{\sqrt{t}} - \sqrt{t}$, then $\dfrac{\mathrm{d}^3 f}{\mathrm{d}t^3}$ is equal to

   a) $\dfrac{-3}{8} t^{-7/2}\,(t+5)$
   b) $\dfrac{3}{8} t^{-7/2}\,(t+3)$
   c) $\dfrac{1}{4} t^{-5/2}\,(t+5)$
   d) $\dfrac{-1}{4} t^{-5/2}\,(3-t)$
   e) $\dfrac{3}{4} t^{-7/2}\,(t-5)$

3. Using the graph of f below, decide which one of the following inequalities is **TRUE**.



a) $f'(0) < f'(b) < f'(e)$
b) $f'(-3) < f'(b) < f'(0)$
c) $f'(a) < f'(c) < f'(e)$
d) $f'(2) < f'(d) < f'(c)$
e) $f'(-2) < f'(1) < f'(d)$

4. If $f(x) = \begin{cases} -x & , \quad x \le 0 \\ \sqrt{x} & , \quad x > 0 \end{cases}$, then which one of the following statements is **FALSE**?

a) the slope of the curve $y = f(x)$ at $x = 0$ is 1
b) f has a vertical tangent at $x = 0$
c) f is continuous at $x = 0$
d) f is not differentiable at $x = 0$
e) the left hand derivative of f at $x = 0$ is -1

5. If $f(x) = (2x+3)^3(x^2-3)^{-2}$, then $f'(1) =$

a) 100

b) 50

c) -25

d) $\dfrac{175}{2}$

e) $\dfrac{-125}{2}$

6. Let $f(x) = \begin{cases} 2-x & , \quad x < 1 \\ x^2 - 2x + 2 & , \quad x \geq 1 \end{cases}$. Then

a) $f'(x) = \begin{cases} -1 & , \quad x < 1 \\ 2x-2 & , \quad x > 1 \end{cases}$

b) $f'(x) = \begin{cases} -1 & , \quad x \leq 1 \\ 2x-2 & , \quad x > 1 \end{cases}$

c) $f'(x) = \begin{cases} -1 & , \quad x < 1 \\ 2x-2 & , \quad x \geq 1 \end{cases}$

d) $f'(x)$ exists for all x except $x = \dfrac{1}{2}$

e) $f'(x)$ does not exist

7. If the position in meters of a body moving along the s-axis is $s = t^3 - 12t^2 + 45t$ in the time interval $[0, 10]$, then the time interval(s) where the particle is moving forward is (are)

   a) $(0, 3) \cup (5, 10)$
   b) $(0, 4)$
   c) $(4, 10)$
   d) $(3, 5)$
   e) $(0, 3) \cup (4, 10)$

8. The value of the limit $\lim\limits_{\theta \to \pi/6} \left(\dfrac{\cot\theta - \sqrt{3}}{\theta - \pi/6} \right)$ is equal to

   a) -4
   b) -2
   c) 6
   d) -8
   e) 3

9. Let $f(x) = \ln(\cos x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. If c satisfies $2f'(c) + f''(c) = 0$, then $f(c)$ is equal to

a) $\frac{-\ln 2}{2}$
b) $-\ln 2$
c) $\ln 2$
d) $\frac{\ln 2}{2}$
e) 0

10. If $F(x) = f(g(x^2))$ where $g(4) = 2$, $g'(4) = -3$, $f'(4) = -6$, and $f'(2) = -2$, then $F'(2) =$

a) 24
b) -8
c) 6
d) -12
e) -36

11. The slope of the tangent line to the curve $y = \ln\left(\dfrac{\sin^{-1} x}{\pi x}\right)$ at $x = \dfrac{1}{2}$ is

a) $\dfrac{4\sqrt{3}}{\pi} - 2$
b) $\dfrac{4}{\pi} - 2\pi$
c) $\dfrac{4\sqrt{3}}{3\pi} - 2$
d) $\dfrac{4}{5\pi} - 2\pi$
e) $\dfrac{4}{\pi} - \sqrt{3}$

12. The equation of the normal line to the curve $(x^2+y^2) = (x-y)^2$ at the point $(1, -1)$ is

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a) $x + y = 0$
b) $2x + y + 1 = 0$
c) $y - 2x = 0$
d) $x + 2y = 0$
e) $x - y - 1 = 0$

13. Let $xy + y^2 = 1$. $\dfrac{d^2y}{dx^2}$ at $(0, -1)$ is equal to

a) $\dfrac{-1}{4}$
b) $\dfrac{1}{8}$
c) $\dfrac{-1}{2}$
d) 1
e) 0

14. If $f(t) = 3^{\log_2 t} + \log_2\left(3^t\right)$, then $f'(2)$ is equal to

a) $\dfrac{\ln 243}{\ln 4}$
b) $\dfrac{\ln 9}{\ln 2}$
c) $1 + \log_2 9$
d) $\ln\left(\dfrac{4}{3}\right)$
e) $1 + \log_2 6$

15. If $y = (\sin x)^{\csc x}$, then $\dfrac{dy}{dx}$ is equal to

a) $(\sin x)^{\csc x} \csc x \cot x \,(1 - \ln(\sin x))$
b) $-(\sin x)^{\csc x - 1} \csc x \cot x$
c) $(\sin x)^{\csc x} \csc x \cot x \,(1 - \ln(\csc x))$
d) $-(\sin x)^{\csc x} \csc x \cot x \ln(\cos x)$
e) $(\sin x)^{\csc x} \csc x \cot x \ln(\sin x)$

16. The derivative of $y = \cot^{-1} x + \tan^{-1}\left(\dfrac{1}{x}\right)$ is equal to

a) $\dfrac{-2}{1+x^2}$
b) $\dfrac{2x^2}{1+x^2}$
c) 0
d) $\dfrac{-1}{1+x^2}$
e) $\dfrac{1}{1+x^2}$

17. The width of a rectangle is increasing at the rate 2 cm/sec while the diagonal is decreasing at the rate 3 cm/sec. When the width is 4 cm and the diagonal is 5 cm, the rate of change of the area of the rectangle is

a) $\frac{-74}{3}$ cm/sec^2
b) $\frac{-10}{3}$ cm/sec^2
c) $\frac{-2}{3}$ cm/sec^2
d) $\frac{92}{3}$ cm/sec^2
e) $\frac{20}{3}$ cm/sec^2

18. A rock thrown vertically upward from the ground at a velocity of 192 ft/sec reaches a height of $s(t) = 192t - 16t^2$ ft after t seconds. Using the given table of values of s, the total distance travelled by the rock from $t = 3$ to $t = 8$ is

t	3	4	5	6	7	8
$s(t)$	432	512	560	576	560	512

a) 208 ft
b) 80 ft
c) 144 ft
d) 64 ft
e) 276 ft

19. A particle moves along the curve $y = x^3$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 3 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 2$ m?

a) $\frac{12}{17}$
b) $\frac{6}{17}$
c) $\frac{-3}{17}$
d) $\frac{3}{8}$
e) $\frac{-2}{27}$

20. The rate of change of $f(x) = \dfrac{x^2 e^{\sqrt{x-1}}}{1-x}$ with respect to x at $x = 2$ is

[Hint: You may use logarithmic differentiation.]

a) $-2e$
b) $8e$
c) $10e$
d) $-6e$
e) $5e$

Q	MM	V1	V2	V3	V4
1	a	e	a	d	e
2	a	e	b	c	b
3	a	d	c	a	c
4	a	d	e	a	e
5	a	b	b	e	c
6	a	c	e	e	b
7	a	e	b	e	e
8	a	e	c	b	e
9	a	c	c	a	c
10	a	a	a	e	b
11	a	c	a	a	a
12	a	b	b	e	d
13	a	e	d	c	a
14	a	b	c	a	a
15	a	a	e	b	d
16	a	b	c	d	b
17	a	b	d	c	d
18	a	e	e	b	c
19	a	b	d	c	d
20	a	c	c	b	b