King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 - Calculus I
Exam I
Term (121)

Tuesday, October 2, 2012 **Time Allowed: 2 hours**

Name: _____ ID Number: _____

Section Number: _____ Serial Number: _____

Class Time: _____ Instructor's Name: _____

Instructions:

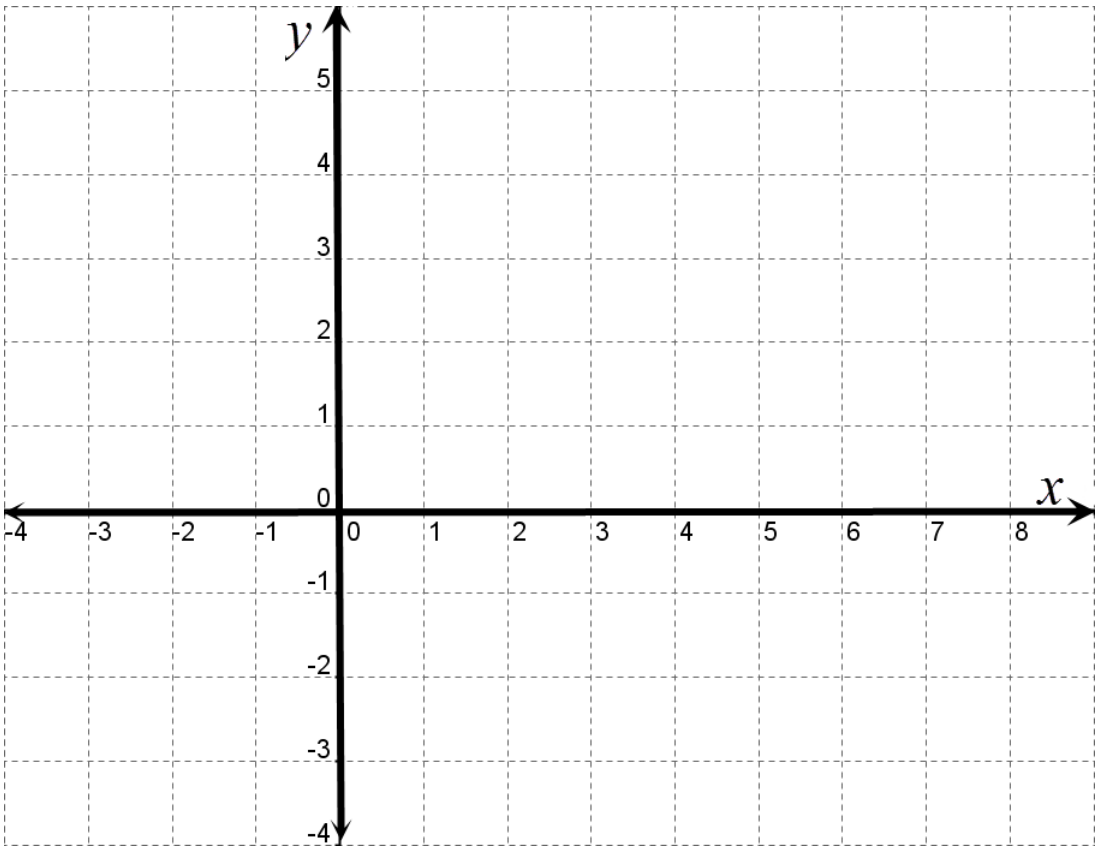
1. Calculators and Mobiles are not allowed.
2. Write neatly and eligibly. You may lose points for messy work.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 7 pages of problems (Total of 10 Problems)

	Points	Maximum Points
Page 1		20
Page 2		13
Page 3		12
Page 4		17
Page 5		22
Page 6		10
Page 7		6
Total		100

1. (2 points) Find the average rate of change of the function $f(x) = x^3 + 1$ over the interval $[-1, 1]$.

2. (12 points) Sketch the graph of a function f that satisfies the following conditions:

   (i) $\lim_{x\to\pm\infty} f(x) = 1$, (ii) $\lim_{x\to 3^-} f(x) = \infty$, (iii) $\lim_{x\to 3^+} f(x) = -\infty$,
   (iv) f has a removable discontinuity at 1, (v) f has a jump discontinuity at 5.



3. (6 points) Using the Sandwich Theorem, show that if $\lim_{x\to 1} |f(x)| = 0$, then $\lim_{x\to 1} f(x) = 0$.

4. Evaluate the limit or show that it does not exist.

i) (4 points) $\lim_{x\to 2^-} \dfrac{|x-2|(x-4)}{(x-2)}$

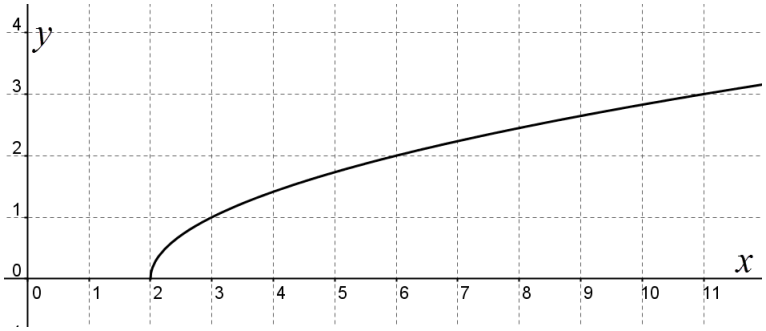
ii) (6 points) $\lim_{\theta\to 0} \tan 3\theta \cdot \csc\theta$

iii) (3 points) $\lim_{x\to 1} \left[\dfrac{1}{x}\right]$, where $[y]$ is the greatest integer less than or equal to y.

iv) (6 points) $\lim_{x\to 3} \dfrac{x^2-9}{\sqrt{x^2+7}-4}$

v) (6 points) $\lim_{x\to 0} \dfrac{x - x\cos x}{\sin^2 x}$

5. (9 points) Use the graph of $f(x) = \sqrt{x-2}$ to find $\delta > 0$ such that if $0 < |x-6| < \delta$, then $|f(x) - 2| < 1$



6. (8 points) Use the Intermediate Value Theorem to prove that the equation $x^3 - 3x - 1 = 0$ has a solution.

7. (12 points) Use limits to find the values of a and b for which the function

$$g(x) = \begin{cases} ax + 5b, & x \leq 0 \\ x^2 + a - 3b, & 0 < x \leq 2 \\ 5x - 3, & x > 2 \end{cases}$$

is continuous at every x.

8. (10 points) Use limits to find the horizontal asymptotes of the curve $y = \dfrac{4 - 3x^3}{\sqrt{x^6 + 9}}$

9. (10 points) Use limits to find all asymptotes of the curve $y = \dfrac{x^2+1}{x-1}$.

10. To each of the following statements, give an example to show that the statement is **False:**

(a) (3 points) If $\lim_{x\to 1}(f(x)\cdot g(x))$ exists, then the limit must be $f(1)\cdot g(1)$

(b) (3 points) If f has domain $(0,\infty)$ and has no horizontal asymptotes, then $\lim_{x\to\infty} f(x)=\infty$ or $\lim_{x\to\infty} f(x)=-\infty$