King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 101
Exam 2
Term 102
Monday, April 25, 2011
Net Time Allowed: 120 minutes

MASTER VERSION

1. The slope of the tangent line to the curve $y = 4\sec x + \tan x$ at $x = \dfrac{\pi}{4}$ is equal to

(a) $2 + 4\sqrt{2}$

(b) 2

(c) $1 + \sqrt{2}$

(d) $-4\sqrt{2}$

(e) -6

2. The equation of the **normal line** to the curve $y = 4x^3 - 6\sqrt{x}$ at the point $(1, -2)$ is

(a) $9y + x + 17 = 0$

(b) $y = 9x + 11$

(c) $y + 9x - 11 = 0$

(d) $y = 17x - 9$

(e) $y + 9x - 17 = 0$

3. If $f(x) = \sqrt[3]{x^2} + \dfrac{1}{\sqrt[4]{x}}$, then $f''(1) =$

(a) $\dfrac{13}{144}$

(b) $\dfrac{-3}{56}$

(c) $\dfrac{77}{144}$

(d) $\dfrac{13}{56}$

(e) $\dfrac{101}{144}$

4. If $x^3 + x^2y - 4y^2 = 6$, then $y' =$

(a) $\dfrac{2xy + 3x^2}{8y - x^2}$

(b) $\dfrac{-3x^2}{x^2 - 8y}$

(c) $\dfrac{3x^2 + xy}{8y}$

(d) $\dfrac{xy - 3x^2}{4y - 2x}$

(e) $\dfrac{x + 3xy}{4y^2 + 4x}$

5. The x-coordinates of the points on the curve $y = 3x - \sin x$ at which the tangent line has slope 4 are

(a) $(2n+1)\pi$, n is integer

(b) $n\pi$, n is integer

(c) $\frac{2n+1}{2}\pi$, n is integer

(d) $2n\pi$, n is integer

(e) $\frac{n\pi}{3}$, n is integer

6. The equation of the tangent line to the curve $x^2 + (y-x)^3 = 9$ at $x = 1$ is

(a) $y = \frac{5}{6}x + \frac{13}{6}$

(b) $y = \frac{5}{3}x + \frac{4}{3}$

(c) $y = \frac{3}{6}x - \frac{17}{6}$

(d) $y = \frac{7}{6}x + \frac{11}{6}$

(e) $y = \frac{1}{6}x + \frac{17}{6}$

7. If $f(x) = \dfrac{2x-1}{(x+3)^3}$, then $f'(x) =$

(a) $\dfrac{9-4x}{(x+3)^4}$

(b) $\dfrac{3x+2}{(x+3)^6}$

(c) $\dfrac{7-4x}{(x+3)^5}$

(d) $\dfrac{3+2x}{(x+3)^4}$

(e) $\dfrac{5+4x}{(x+3)^5}$

8. If $y = \sqrt[5]{u^2-3}$ and $u = \sqrt[3]{x^2-1}$, then $\left.\dfrac{dy}{dx}\right|_{x=3} =$

(a) $\dfrac{2}{5}$

(b) $\dfrac{8}{15}$

(c) $\dfrac{8}{\sqrt[3]{2}}$

(d) $\dfrac{3}{\sqrt[5]{4}}$

(e) $\dfrac{1}{5}$

9. $\lim_{t\to 0} \dfrac{\sin^2(3t)}{t^3 - 3t^2} =$

(a) -3

(b) 3

(c) $\dfrac{1}{9}$

(d) $\dfrac{2}{3}$

(e) $\dfrac{4}{27}$

10. Let f be a differentiable function such that $f(2) = 2$, $f(4) = 1$, $f'(2) = 3$, and $f'(4) = -1$. If $G(x) = f(2x) \cdot f(x)$, then $G'(2) =$

(a) -1

(b) 1

(c) 0

(d) 5

(e) 3

11. If $h(x) = \ln(x + 2\ln x)$, then $h'(x) =$

(a) $\dfrac{x+2}{x^2 + 2x\ln x}$

(b) $\dfrac{x+2}{x + 2\ln x}$

(c) $\dfrac{x+1}{x^2 + 2\ln x}$

(d) $\dfrac{1}{x + 2\ln x}$

(e) $\dfrac{2}{x(x + 2\ln x)}$

12. If $f(t) = 3^{\sin^2(3t)}$, then $f'(t) =$

(a) $3^{\sin^2(3t)} \cdot \ln(27) \cdot \sin(6t)$

(b) $3^{\sin^2(3t)} \cdot \ln 3 \cdot 2\sin(3t)$

(c) $3^{\sin^2(3t)} \cdot \ln 9 \cdot 3\cos(3t)$

(d) $\sin^2(3t) \cdot 3^{\sin^2(3t)-1}$

(e) $3^{\sin^2(3t)} \cdot \ln 3 \cdot 3\sin(3t) \cdot \cos(3t)$

13. If $y = x^{\sin^{-1} x}$, then $\dfrac{y'}{y} =$

(a) $\dfrac{\sin^{-1} x}{x} + \dfrac{\ln x}{\sqrt{1-x^2}}$

(b) $\dfrac{1}{x\sqrt{1-x^2}}$

(c) $\dfrac{\sin^{-1} x}{\sqrt{1-x^2}} + \dfrac{\ln x}{x}$

(d) $(\sin^{-1} x) \ln x$

(e) $x^{\sin^{-1} x} \cdot \ln x$

14. $\lim\limits_{x \to 0} \dfrac{\cos x + \sin(2x) - 1}{\tan x} =$

(a) 2

(b) 1

(c) 0

(d) -3

(e) Does not exist

15. If $f(x) = \dfrac{\sqrt[3]{3x-2}}{e^{x^2}(x^3+1)^{10}}$, then $f'(1) =$
[Hint: you may use logarithmic differentiation]

(a) $\dfrac{-1}{64e}$

(b) $\dfrac{32}{e}$

(c) $\dfrac{e}{15}$

(d) $\dfrac{-16}{5e}$

(e) $\dfrac{1}{16e}$

16. If $y = \tan^{-1}\left(\dfrac{1+x}{1-x}\right)$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{1}{1+x^2}$

(b) $\dfrac{2}{(1+x)^2}$

(c) $\dfrac{(1-x)^2}{1-(1+x)^2}$

(d) $\dfrac{(1+x)^2}{(1-x)^2}$

(e) $\dfrac{2}{2+x^2}$

17. A parabola $y = ax^2 + bx + c$ passes through the point $(1, 7)$, has a tangent line at $x = -1$ with slope 6, and has a tangent line at $x = 5$ with slope -2. The value of $6a + 3b + c$ is equal to

(a) 13

(b) -12

(c) 7

(d) -8

(e) 0

18. If $y = mx + k$ is the equation of a line parallel to the line $y = x$ and tangent to the graph of $y = e^{x+2}$, then $m + k =$

(a) 4

(b) -2

(c) 2

(d) 3

(e) 5

19. A particle moves according to the law of motion

$$f(t) = 9te^{-t/3}, \quad 0 \leq t \leq 8.$$

The time interval(s) on which the particle is **slowing down** is(are)

(a) $(0,3)$ and $(6,8)$

(b) $(3,6)$

(c) $(0,2)$ and $(3,6)$

(d) $(2,6)$

(e) $(0,3)$ and $(4,8)$

20. The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. When the length of an edge is 30 cm, the **surface area** of the cube is increasing at a rate of

(a) $\frac{4}{3}$ cm^2/min

(b) $\frac{5}{9}$ cm^2/min

(c) $6 \text{ cm}^2/\text{min}$

(d) $\frac{5}{3}$ cm^2/min

(e) $5 \text{ cm}^2/\text{min}$