

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Group Quiz 1
Semester 101

Group Names:	ID:	Signature:
1)		
2)		
3)		
4)		

Group Quiz Objectives:

- How to solve math problems from sec 2.3.
- How to write correct math solutions (in each problem, we provide you with an example to show you how to write a correct solution).

Group Quiz Guidelines:

- Please print out this file, solve together, and submit – in class – on **Monday 11th October, 2010**.
- The mark is given to each member of the group.

() Say Bismillah & Good luck ()

1. Evaluate the limit if it exists. (read examples 3, 6, 7, 10 on pages 102, 103, 104, 105).

$$\begin{aligned}
 \text{(a)} \lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4} &= \lim_{x \rightarrow 4} \frac{\cancel{x(x-4)}}{(x+1)\cancel{(x-4)}} \quad x \neq 4 \\
 &= \lim_{x \rightarrow 4} \frac{x}{x+1} \\
 &= \frac{4}{5} \quad (\text{by direct substitution})
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \lim_{x \rightarrow 1} \frac{\sqrt{x^2+1} - \sqrt{2}}{1-x} &= \lim_{x \rightarrow 1} \frac{\sqrt{x^2+1} - \sqrt{2}}{1-x} \times \frac{\sqrt{x^2+1} + \sqrt{2}}{\sqrt{x^2+1} + \sqrt{2}} \\
 &= \lim_{x \rightarrow 1} \frac{(x^2+1) - 2}{(1-x)(\sqrt{x^2+1} + \sqrt{2})} \\
 &= \lim_{x \rightarrow 1} \frac{(x^2-1)}{(1-x)(\sqrt{x^2+1} + \sqrt{2})} \\
 &= \lim_{x \rightarrow 1} \frac{-(1-x)(1+x)}{(1-x)(\sqrt{x^2+1} + \sqrt{2})} \quad x \neq 1 \\
 &= \lim_{x \rightarrow 1} \frac{-(1+x)}{\sqrt{x^2+1} + \sqrt{2}} \\
 &= \frac{-2}{2\sqrt{2}} \quad (\text{by direct subs}) \\
 &= -\frac{1}{\sqrt{2}}
 \end{aligned}$$

$$(c) \lim_{x \rightarrow 0^+} \frac{3}{x} \left(\frac{1}{4+x} - \frac{1}{4-x} \right) = \lim_{x \rightarrow 0^+} \frac{3}{x} \left(\frac{(4-x) - (4+x)}{(4+x)(4-x)} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{3}{x} \left(\frac{-2x}{(4+x)(4-x)} \right) \quad x \neq 0$$

$$= \lim_{x \rightarrow 0^+} \frac{-6}{(4+x)(4-x)}$$

$$= \frac{-6}{16} =$$

$$= -3/8$$

(by direct subs)

$$(d) \lim_{x \rightarrow 2^-} ([x-1] - x^2), \text{ where } [\cdot] \text{ denotes the greatest integer function.}$$

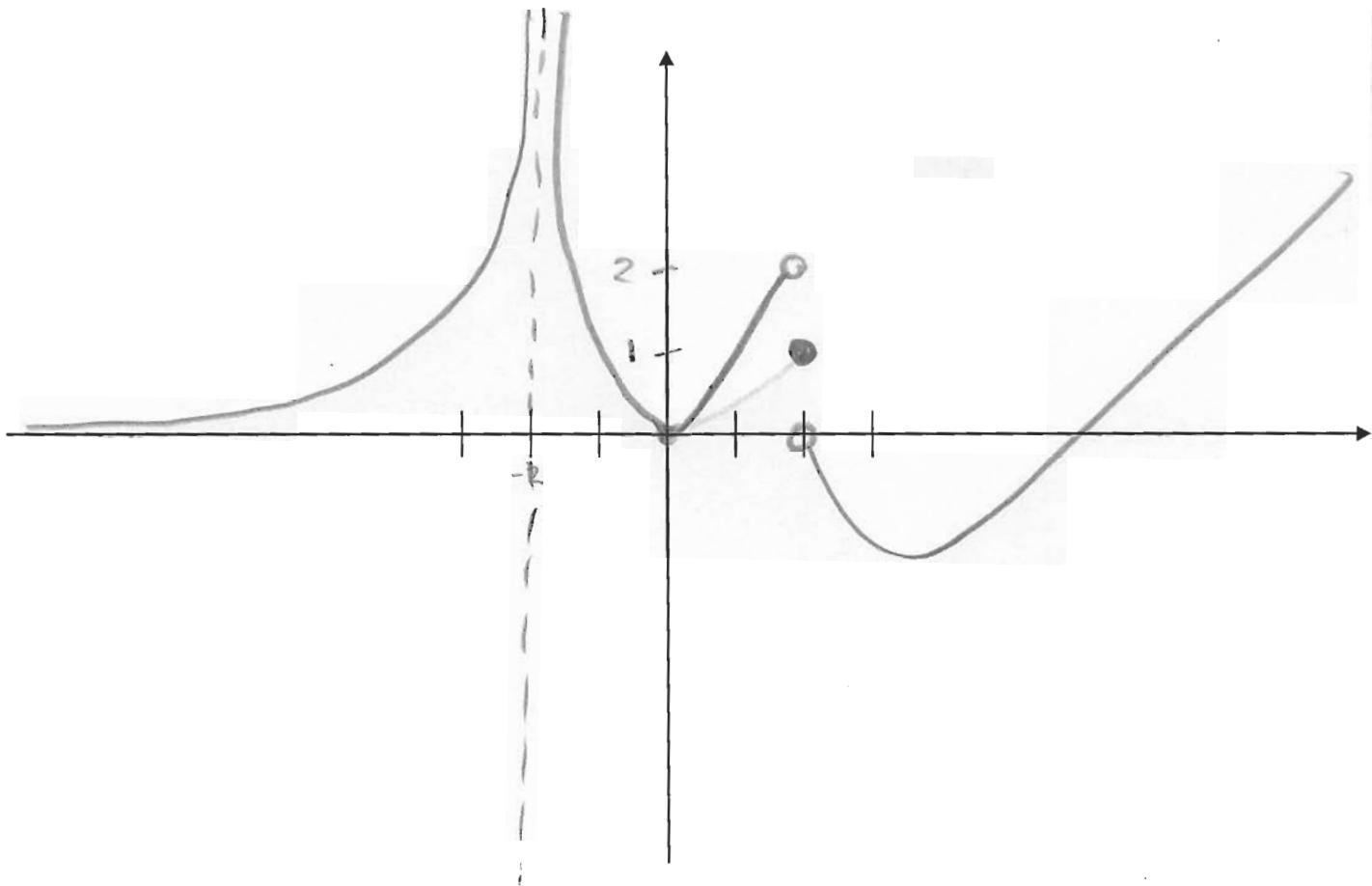
since, $[\lfloor x-1 \rfloor] = 0$ for $1 \leq x < 2$, we have

$$\lim_{x \rightarrow 2^-} [\lfloor x-1 \rfloor] = 0 \quad \text{--- (1)}$$

$$\begin{aligned} \text{Now, } \lim_{x \rightarrow 2^-} ([\lfloor x-1 \rfloor] - x^2) &= \lim_{x \rightarrow 2^-} [\lfloor x-1 \rfloor] - \lim_{x \rightarrow 2^-} x^2 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

2. Sketch the graph of a function f that satisfies all of the given conditions:

$$f(0) = 0, f(2) = 1, \lim_{x \rightarrow 2^+} f(x) = 0, \lim_{x \rightarrow 2^-} f(x) = 2, \lim_{x \rightarrow -2} f(x) = +\infty.$$



3. Find the vertical asymptotes of the graph of the function

$$f(x) = \frac{4x-1}{x^3 - 8x^2}. \text{ Explain.} \quad (\text{study the infinite limit}).$$

$$f(x) = \frac{4x-1}{x^2(x-8)}$$

Domain of $f(x)$: $\mathbb{R} - \{0, 8\}$

We have $\lim_{x \rightarrow 0^+} \frac{1}{x^2} = \infty$ and $\lim_{x \rightarrow 0^-} \frac{1}{x^2} = \infty$

$$\lim_{x \rightarrow 0} f(x) = \left(\lim_{x \rightarrow 0} \frac{1}{x^2} \right) \left(\lim_{x \rightarrow 0} \frac{4x-1}{x-8} \right) \\ = \infty$$

Hence, $x = 0$ is a vertical asymptote

$$\lim_{x \rightarrow 8^+} \frac{1}{x-8} = +\infty \quad \text{and} \quad \lim_{x \rightarrow 8^-} \frac{1}{x-8} = -\infty$$

$$\lim_{x \rightarrow 8^+} f(x) = \left(\lim_{x \rightarrow 8^+} \frac{1}{x-8} \right) \left(\lim_{x \rightarrow 8^+} \frac{4x-1}{x^2} \right) = +\infty$$

$$\lim_{x \rightarrow 8^-} f(x) = \left(\lim_{x \rightarrow 8^-} \frac{1}{x-8} \right) \left(\lim_{x \rightarrow 8^-} \frac{4x-1}{x^2} \right) = -\infty$$

Hence, $x = 8$ is also a vertical asymptote

4. Evaluate the limit if it exists. (read example 11 on page 106).

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x}$$

let $x \in (-1, 1)$

Since, $-1 \leq \sin \frac{\pi}{x} \leq 1$

Multiply by $\sqrt{x^3 + x^2}$

$$-\sqrt{x^3 + x^2} \leq \sqrt{x^3 + x^2} \sin \frac{\pi}{x} \leq \sqrt{x^3 + x^2}$$

We have,

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} = 0 \quad \text{and} \quad \lim_{x \rightarrow 0} (-\sqrt{x^3 + x^2}) = 0$$

Taking $f(x) = -\sqrt{x^3 + x^2}$, $g(x) = \sqrt{x^3 + x^2} \sin \frac{\pi}{x}$

and $h(x) = \sqrt{x^3 + x^2}$ in the squeeze

Theorem, we obtain

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0$$

5. If $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$. Find $\lim_{x \rightarrow -2} \frac{f(x)}{x}$

$$\begin{aligned} 1 &= \lim_{x \rightarrow -2} \frac{f(x)}{x^2} = \lim_{x \rightarrow -2} \left(\frac{1}{x}\right) \left(\frac{f(x)}{x}\right) \\ &= \left(\lim_{x \rightarrow -2} \frac{1}{x}\right) \left(\lim_{x \rightarrow -2} \frac{f(x)}{x}\right) \\ &= -\frac{1}{2} \left(\lim_{x \rightarrow -2} \frac{f(x)}{x}\right) \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow -2} \frac{f(x)}{x} = -2$$

6. If $\lim_{x \rightarrow 2} \frac{f(x)-5}{x-2} = 3$. Find $\lim_{x \rightarrow 2} (2f(x)-3)$

$$\text{Since, } \lim_{x \rightarrow 2} (x-2) \neq 0 \text{ and } \lim_{x \rightarrow 2} \frac{f(x)-5}{x-2} = 3$$

$$\text{Then we have: } \lim_{x \rightarrow 2} (f(x)-5) = 0$$

which implies

$$\lim_{x \rightarrow 2} f(x) - \lim_{x \rightarrow 2} 5 = 0$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = 5$$

$$\text{Now, } \lim_{x \rightarrow 2} (2f(x)-3) = 2 \lim_{x \rightarrow 2} f(x) - 3 = 2(5) - 3 = 7$$