

1) Find $\lim_{n \rightarrow \infty} (\sqrt[n^{200} + n^{100} + 1] - n^{100})$

2) Find $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$

3) Find $\lim_{x \rightarrow \infty} [(x^6 + x^5)^{1/6} - (x^6 - x^5)^{1/6}]$

4) Find $\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n^2}}{\ln n}$

5) Find a function $f(x)$ such that $f'(x^3) = 3x^2$

6) Let $f(x) = \sin^4 x + \cos^4 x$
Find $f^{(n)}(\frac{\pi}{3})$

7) Let $f(x) = \frac{x^n}{1-x}$
Find a formula for $f^{(n)}(x)$

8) Find all discontinuity points of the function $f(x) = \begin{cases} 0 & \text{if } x \text{ rational} \\ x & \text{if } x \text{ is not rational} \end{cases}$

9) Find the global maximum of $f(x) = \frac{1}{1+|x-2|} + \frac{1}{1+|x+6|}$

10) Find a function f such that $f(1) = -1$, $f(4) = 7$, and $f'(x) > 3$ for all x .

11) Are there two functions f and g that have the property that:

$$(fg)' = f' \cdot g'$$

$$\left(\frac{f}{g}\right)' = f'/g'$$

We require that the functions to have derivatives at every point, also $g \neq 0$, $g' \neq 0$

12) Let $f(x)$ be a continuous function defined on $(-\infty, +\infty)$ with the following prop:

- $f(1000) = 999$
- $f(x) \cdot f(f(x)) = 1$

Find $f(101)$

13) Find two functions f and g that are not constant multiples of each other that satisfy $f' = g$ and $g' = f$