King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 092
Sunday 13/06/2010
Net Time Allowed: 180 minutes

MASTER VERSION

1. An equation of the tangent line to the curve $y = x^2 \ln x$ when $x = 1$ is given by

(a) $y = x - 1$

(b) $y = 3x - 3$

(c) $y = x$

(d) $y = \frac{1}{2}x - \frac{1}{2}$

(e) $y = 2 - 2x$

2. If the line $y = Ax + B$ is a slant asymptote of the curve

$$y = \frac{4x^3 - 6x^2 + 1}{x^2 - 2x + 3},$$

then $A - 2B =$

(a) 0

(b) 1

(c) -7

(d) 6

(e) -10

3. Newton's Method is used to find an approximation to one of the real roots of the equation

$$x^5 = x + 1.$$

If $x_1 = 1$ is the first approximation, then the second approximation $x_2 =$

(a) $\frac{5}{4}$

(b) $\frac{3}{7}$

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{1}{3}$

4. The linear approximation of $f(x) = e^{\sin x}$ at $a = 0$ is given by

(a) $e^{\sin x} \approx 1 + x$

(b) $e^{\sin x} \approx 1 - x$

(c) $e^{\sin x} \approx 1 + 2x$

(d) $e^{\sin x} \approx 2 + x$

(e) $e^{\sin x} \approx \frac{1}{2} - x$

5. If $g(x) = \cosh^5(2x)$, then $g'(x) =$

(a) $10 \sinh(2x) \,.\, \cosh^4(2x)$

(b) $-10 \cosh^4(2x)$

(c) $-5 \sinh(2x) \,.\, \cosh^4(2x)$

(d) $5 \cosh^4(2x)$

(e) $20 \sinh(2x) \,.\, \cosh^4(2x)$

6. The function $f(x) = \dfrac{1}{1 - \sin(2x)}$ is continuous everywhere <u>except</u> at the numbers

(a) $\dfrac{\pi}{4} + k\pi$, k is integer

(b) $2k\pi$, k is integer

(c) $\dfrac{\pi}{3} + 2k\pi$, k is integer

(d) $\dfrac{\pi}{2} + k\pi$, k is integer

(e) $\dfrac{2\pi}{3} + k\pi$, k is integer

7. The function $f(x) = |2x+3|$ has

(a) one critical number

(b) two critical numbers

(c) three critical numbers

(d) four critical numbers

(e) no critical numbers

8. The slope of the tangent line to the graph of

$$\pi xy = 8\tan^{-1}\left(\frac{2y}{x}\right)$$

at the point $(2,1)$ is

(a) $\dfrac{2+\pi}{4-2\pi}$

(b) $\dfrac{4-\pi}{2-\pi}$

(c) $\dfrac{1+2\pi}{4-2\pi}$

(d) $\dfrac{2+\pi}{1-4\pi}$

(e) $\dfrac{16}{\pi}$

9. $\lim_{x \to 0} \dfrac{x^2 - \tan^{-1} x}{x \cos x} =$

(a) -1

(b) 3

(c) $+\infty$,

(d) 2

(e) $-\dfrac{1}{2}$

10. If $y = \dfrac{x+1}{x-1}$, then $y''' + 3(y')^2 =$

(a) 0

(b) $\dfrac{12}{(x-1)^4}$

(c) $\dfrac{8}{(x-1)^4}$

(d) $\dfrac{20}{(x-1)^3}$

(e) $\dfrac{-10}{(x-1)^4}$

11. The value(s) of c satisfying the conclusion of Rolle's Theorem for $f(x) = \sqrt{x} - \frac{1}{4}x$ on the interval $[0, 16]$ is (are)

(a) 4

(b) ± 4

(c) 0 and 4

(d) 1

(e) ± 2

12. If $y = (\sqrt{x})^{\tan x}$, then $y' =$

(a) $y\left[\dfrac{\tan x}{2x} + \sec^2 x \,.\, \ln\sqrt{x}\right]$

(b) $y\left[\dfrac{\tan x}{x} + \sec^2 x\right]$

(c) $y\ \tan x \,.\, \ln\sqrt{x}$

(d) $\dfrac{\tan x\,.(\sqrt{x})^{\tan x - 1}}{2\sqrt{x}}$

(e) $y\left[\tan x + \sec^2 x \,.\, \ln\ x\right]$

13. The function

$$f(x) = \begin{cases} x+1, & \text{if } x \leq 1 \\ \dfrac{1}{x} & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$

is continuous on

(a) $(-\infty, 1) \cup (1, 3) \cup (3, +\infty)$

(b) $(-\infty, 0) \cup (0, 1) \cup (1, +\infty)$

(c) $(-\infty, 3) \cup (3, +\infty)$

(d) $(-\infty, +\infty)$

(e) $[3, +\infty)$

14. If M and m are respectively the absolute maximum and absolute minimum values of $f(x) = x^4 - 4x^2 + 2$ on the interval $[-2, 3]$, then $M + m =$

(a) 45

(b) 49

(c) 50

(d) 41

(e) 53

15. $\lim\limits_{x \to -1^-} \dfrac{x}{x^2 - 1} =$

(a) $-\infty$

(b) $+\infty$

(c) $-\dfrac{1}{2}$

(d) 0

(e) -1

16. The most general antiderivative of the function

$$f(x) = \frac{3x^{1/4} - x + x^2 e^x}{x^2}$$

is

(a) $-4\,x^{-3/4} - \ln|x| + e^x + C$

(b) $x^{-3/4} - \dfrac{1}{x^2} + e^x + C$

(c) $-4\,x^{-3/4} - \ln|x| + \dfrac{e^{x+1}}{x+1} + C$

(d) $3\,x^{-5/4} + \ln|x| + e^x + C$

(e) $-\dfrac{4}{9}\,x^{-7/4} - \dfrac{1}{x^2} + e^x + C$

17. Let $y = x^4 + 5x^2 - 2$. Using differentials, the change in y when x changes from 1 to 1.001 is approximately equal to

(a) 0.014

(b) 0.001

(c) 0.01

(d) 0.021

(e) 0.045

18. If $y = (u^2 - 1)^3$ and $u = \sqrt[3]{1+x}$, then $\frac{dy}{dx}|_{x=7} =$

(a) 9

(b) 12

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

(e) 6

19. $\lim\limits_{x\to+\infty}\left(1-\dfrac{1}{2x}\right)^{4x} =$

(a) e^{-2}

(b) e^{-6}

(c) $e^{\frac{1}{2}}$

(d) e^{-1}

(e) e

20. $(\cosh x - \sinh x)^{30} + (\cosh x + \sinh x)^{30} =$

(a) $2\cosh(30x)$

(b) $2\sinh(30x)$

(c) e^{30x}

(d) $(2\cosh x)^{30}$

(e) 2

21. The graph of the function $f(x) = 3x^{2/3}\left(\frac{1}{5}x - 1\right)$ is **concave upward** on the interval(s)

(a) $(-1, +\infty)$

(b) $(-\infty, -1)$ and $(2, +\infty)$

(c) $(-\infty, -1)$

(d) $(-\infty, 0)$ and $(2, +\infty)$

(e) $(-1, 2)$

22. If a particle is moving according to the following data

$$a(t) = 6t - \sqrt{t} \quad , v(1) = \frac{1}{3} \quad , s(1) = \frac{-4}{15},$$

then $s(2) =$

(a) $5 - \frac{16}{15}\sqrt{2}$

(b) $15 - 4\sqrt{2}$

(c) $6 + \frac{4}{15}\sqrt{2}$

(d) $8 - \frac{16}{15}\sqrt{2}$

(e) $9 + \frac{2}{15}\sqrt{2}$

23. The function $f(x) = 2x - \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

(a) has a local maximum at$x = \frac{\pi}{4}$

(b) has a local maximum at$x = -\frac{\pi}{4}$

(c) has a local minimum at$x = \frac{\pi}{3}$

(d) has a local minimum at $x = \frac{\pi}{4}$

(e) has no local minimum at any point in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

24. The function $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$ is **increasing** on the interval(s)

(a) $(-2, 0)$

(b) $(-2, +\infty)$

(c) $(-\infty, 0)$ and $(2, \infty)$

(d) $(-\infty, -2)$

(e) $(-\infty, -2)$ and $(0, +\infty)$

25. A street light is located at the top of a 7-meter-tall pole. A man $2\,m$ tall walks away from the pole with a speed $3\,m/s$ along a straight path. When the man is $15\,m$ from the pole, **the tip of his shadow** is moving at a rate of

(a) $\frac{21}{5}\ m/s$

(b) $3\ m/s$

(c) $\frac{7}{5}\ m/s$

(d) $\frac{2}{7}\ m/s$

(e) $\frac{4}{7}\ m/s$

26. The area of the largest rectangle that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$ is

(a) $12\sqrt{3}$

(b) $\frac{81}{4}$

(c) $2\sqrt{3}$

(d) 9

(e) $4\sqrt{3}$

27. If $f(1) = 2$ and $f'(x) \geq 5$ for $1 \leq x \leq 4$, then the smallest possible value that $f(4)$ can have is (Hint: Use the Mean Value Theorem)

(a) 17

(b) 10

(c) -15

(d) 2

(e) 5

28. If the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$ at the point (a, b), then $ac + b =$

(a) 36

(b) -12

(c) 12

(d) 24

(e) -36

21. The area of the largest rectangle that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$ is

(a) 9

(b) $4\sqrt{3}$

(c) $12\sqrt{3}$

(d) $2\sqrt{3}$

(e) $\dfrac{81}{4}$

22. The function $f(x) = 2x - \tan x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

(a) has a local minimum at$x = \dfrac{\pi}{3}$

(b) has a local maximum at$x = \dfrac{\pi}{4}$

(c) has a local maximum at$x = -\dfrac{\pi}{4}$

(d) has no local minimum at any point in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

(e) has a local minimum at $x = \dfrac{\pi}{4}$

23. A street light is located at the top of a 7-meter-tall pole. A man $2\,m$ tall walks away from the pole with a speed $3\,m/s$ along a straight path. When the man is $15\,m$ from the pole, **the tip of his shadow** is moving at a rate of

(a) $\frac{21}{5}\ m/s$

(b) $3\ m/s$

(c) $\frac{4}{7}\ m/s$

(d) $\frac{2}{7}\ m/s$

(e) $\frac{7}{5}\ m/s$

24. The graph of the function $f(x) = 3x^{2/3}\left(\frac{1}{5}x - 1\right)$ is **concave upward** on the interval(s)

(a) $(-\infty, -1)$

(b) $(-\infty, 0)$ and $(2, +\infty)$

(c) $(-1, +\infty)$

(d) $(-\infty, -1)$ and $(2, +\infty)$

(e) $(-1, 2)$

25. The function $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$ is **increasing** on the interval(s)

(a) $(-\infty, -2)$

(b) $(-\infty, -2)$ and $(0, +\infty)$

(c) $(-\infty, 0)$ and $(2, \infty)$

(d) $(-2, 0)$

(e) $(-2, +\infty)$

26. $(\cosh x - \sinh x)^{30} + (\cosh x + \sinh x)^{30} =$

(a) 2

(b) $(2\cosh x)^{30}$

(c) e^{30x}

(d) $2\cosh(30x)$

(e) $2\sinh(30x)$

27. If a particle is moving according to the following data

$$a(t) = 6t - \sqrt{t} \quad , v(1) = \frac{1}{3} \quad , s(1) = \frac{-4}{15},$$

then $s(2) =$

(a) $6 + \frac{4}{15}\sqrt{2}$

(b) $15 - 4\sqrt{2}$

(c) $8 - \frac{16}{15}\sqrt{2}$

(d) $9 + \frac{2}{15}\sqrt{2}$

(e) $5 - \frac{16}{15}\sqrt{2}$

28. If $f(1) = 2$ and $f'(x) \geq 5$ for $1 \leq x \leq 4$, then the smallest possible value that $f(4)$ can have is (Hint: Use the Mean Value Theorem)

(a) 17

(b) 10

(c) -15

(d) 2

(e) 5

Q	MM	V1	V2	V3	V4
1	a	b	c	b	b
2	a	a	e	c	b
3	a	e	c	e	d
4	a	c	a	a	e
5	a	d	c	e	e
6	a	e	b	e	e
7	a	e	a	e	a
8	a	c	c	b	a
9	a	e	c	d	c
10	a	b	b	d	d
11	a	c	d	e	b
12	a	a	c	b	e
13	a	b	a	d	b
14	a	d	d	c	b
15	a	a	e	d	d
16	a	c	e	a	b
17	a	b	d	a	d
18	a	e	e	d	e
19	a	e	d	d	d
20	a	d	d	b	d
21	a	b	c	b	c
22	a	a	e	a	b
23	a	b	b	b	a
24	a	d	c	c	c
25	a	b	a	b	d
26	a	d	d	e	d
27	a	a	d	a	e
28	a	a	c	b	a