

King Fahd University of Petroleum and Minerals
College of Sciences
Quiz #4(B)

St. ID: _____ St. Name: _____ Section: _____ Serial#: _____

Q1: Roll a die twice. Then:

- a) find the probability that the sum of the two numbers is at least 4.

Solution:

$$\begin{aligned} P(\text{that the sum of the two numbers is at least 4}) &= 1 - P(\text{that the sum of the two numbers is less than 4}) \\ &= 1 - P(\text{that the sum of the two numbers is either 2 or 3}) \\ &= 1 - P\{(1,1), (1,2), (2,1)\} = 1 - \frac{3}{36} = \frac{11}{12} \end{aligned}$$

- b) find the probability that the sum of the two numbers is at least 4 given that their sum is at most 6.

Solution:

$$\begin{aligned} P(\text{that the sum of the two numbers is at least 4 given that their sum is at most 6}) &= P(A/B) = P(A \cap B)/P(B) \\ &= P(\text{the sum of the two numbers is 4 or 5 or 6})/P(\text{the sum of the two numbers is at most 6}) \\ &= \left(\frac{12}{36}\right)/\left(\frac{15}{36}\right) = \frac{12}{15} = 0.8 \end{aligned}$$

- c) Let A denote the event where the sum of the numbers is between 6 and 8 and B denote the event where the number on the number on the 2nd roll is either 4 or 6. Then are A and B independent? Why?

Solution:

$$P(A \cap B) = P\{(2,4), (3,4), (4,4), (1,6), (2,6)\} = \frac{5}{36}$$

$$P(A) = \frac{16}{36} \quad \text{and} \quad P(B) = \frac{12}{36}$$

imply that $P(A \cap B) \neq P(A)P(B)$.

Thus A and B are **not** independent.

Q2: Draw 5 balls from an urn containing 10 white balls and 15 black balls.
Then find the probability that you will get 3 black balls if drawing is :
a) with replacement

Solution:

$$\begin{aligned} & P(\text{you will get 3 black balls}) \\ &= ({}^5C_3)P(\text{bbbww}) = ({}^5C_3)\left(\frac{15}{25}\right)\left(\frac{15}{25}\right)\left(\frac{15}{25}\right)\left(\frac{10}{25}\right)\left(\frac{10}{25}\right) \end{aligned}$$

b) without replacement

Solution:

$$\begin{aligned} & P(\text{you will get 3 black balls}) \\ &= ({}^5C_3)P(\text{bbbww}) = ({}^5C_3)\left(\frac{15}{25}\right)\left(\frac{14}{24}\right)\left(\frac{13}{23}\right)\left(\frac{10}{22}\right)\left(\frac{9}{21}\right) \end{aligned}$$

Or

$$P(\text{you will get 3 black balls}) = \frac{({}^{15}C_3)({}^{10}C_2)}{{}^{25}C_5}$$

Q3: Given that $P(A) = .38$, $P(B) = .32$ and $P(A \cup B) = .64$, then
a) find $P(A' / B')$

$$\text{Solution: } P(A' / B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - .64}{1 - .32} = \frac{.36}{.68}$$

b) Are A and B independent? Why?

Solution: No, because $P(A' / B') \neq P(A')$

Or because $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

implies that $P(A \cap B) = .38 + .32 - .64 = .06 \neq P(A)P(B) = (.38)(.32)$