

King Fahd University of Petroleum and Minerals
College of Sciences
Quiz #4(A)

St. ID: _____ St. Name: _____ Section: _____ Serial#: _____

Q1: Roll a die twice. Then:

- a) find the probability that the sum of the two numbers is at least 4.

Solution:

$$\begin{aligned} P(\text{that the sum of the two numbers is at least 4}) &= 1 - P(\text{that the sum of the two numbers is less than 4}) \\ &= 1 - P(\text{that the sum of the two numbers is either 2 or 3}) \\ &= 1 - P\{(1,1), (1,2), (2,1)\} = 1 - \frac{3}{36} = \frac{11}{12} \end{aligned}$$

- b) find the probability that the sum of the two numbers is at least 4 given that their sum is at most 5.

Solution:

$$\begin{aligned} P(\text{that the sum of the two numbers is at least 4 given that their sum is at most 5}) &= P(A/B) = P(A \cap B)/P(B) \\ &= P(\text{the sum of the two numbers is 4 or 5})/P(\text{the sum of the two numbers is at most 5}) \\ &= \left(\frac{7}{36}\right)/\left(\frac{10}{36}\right) = \frac{7}{10} = 0.7 \end{aligned}$$

- c) Let A denote the event where the sum of the numbers is between 5 and 7 and B denote the event where the number on the number on the 2nd roll is either 3 or 5. Then are A and B independent? Why?

Solution:

$$P(A \cap B) = P\{(2,3), (3,3), (4,3), (1,5), (2,5)\} = \frac{5}{36}$$

$$P(A) = \frac{15}{36} \quad \text{and} \quad P(B) = \frac{12}{36}$$

imply that $P(A \cap B) = P(A)P(B)$.

Thus A and B are independent.

Q2: Draw 5 balls from an urn containing 8 white balls and 17 black balls. Then find the probability that you will get 3 black balls if drawing is :

a) with replacement

Solution:

$$\begin{aligned} & P(\text{you will get 3 black balls}) \\ &= ({}^5C_3)P(\text{bbbww}) = ({}^5C_3)\left(\frac{17}{25}\right)\left(\frac{17}{25}\right)\left(\frac{17}{25}\right)\left(\frac{8}{25}\right)\left(\frac{8}{25}\right) \end{aligned}$$

b) without replacement

Solution:

$$\begin{aligned} & P(\text{you will get 3 black balls}) \\ &= ({}^5C_3)P(\text{bbbww}) = ({}^5C_3)\left(\frac{17}{25}\right)\left(\frac{16}{24}\right)\left(\frac{15}{23}\right)\left(\frac{8}{22}\right)\left(\frac{7}{21}\right) \end{aligned}$$

Or

$$P(\text{you will get 3 black balls}) = \frac{({}^{17}C_3)({}^8C_2)}{{}^{25}C_5}$$

Q3: Given that $P(A) = .42$, $P(B) = .35$ and $P(A \cup B) = .75$, then

a) find $P(A' / B')$

Solution:
$$P(A' / B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - .75}{1 - .35} = \frac{.25}{.65}$$

b) Are A and B independent? Why?

Solution: No, because $P(A' / B') \neq P(A')$

Or because $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

implies that $P(A \cap B) = .35 + .42 - .75 = .02 \neq P(A)P(B) = (.35)(.42)$