

King Fahd University of Petroleum and Minerals  
 Department of Mathematics  
**SYLLABUS**  
 Semester II: 2023-2024 (232)

**Instructor:** Dr. A. Bonfoh  
**Course #:** MATH 568  
**Title:** Advanced Partial Differential Equations I

**Textbook:** A basic course in Partial Differential Equations by Y. Qing Han, First Edition.

**References:**  
 1. Partial Differential Equations by L. C. Evans (Second Edition, 2010)  
 2. Beginning Partial Differential Equation. by P. O’Neil. (Second Edition, 2008)

**Objectives:** The course aims to reinforce students knowledge on concepts of existence, uniqueness and properties of solutions to first and second-order linear and quasilinear PDEs. Energy methods to solve nonlinear PDEs will be introduced. Applications to the wave equation, the heat equation and the Laplace equation are considered. In particular, the weak solution and its regularity of the non-homogeneous Laplace IBVP will be considered.

**Course description:** First order linear and nonlinear equations. Classification of Second order equations. The wave equation, heat equation and Laplace’s equation. Green’s functions, conformal mapping. Separation of variables, Sturm-Liouville theory. Maximum principles and regularity theorems.

**Learning outcomes:** Upon successful completion of this course, a student should be able to:

- Solve quasilinear first order equation by the method of characteristics.
- Classify and solve 2<sup>nd</sup> order PDE’s by the method of characteristics.
- Solve the wave equation and analyze the well-posedness.
- Solve IBVP heat equation by using the maximum principle.
- Know the proofs of the representation theorems, Mean value problem, and maximum principles for Laplace equation.
- Apply Green’s function method and method of images to solve the Dirichlet and Neumann problems for the Laplace equation.
- Apply Energy methods to solve 2<sup>nd</sup> order linear and nonlinear PDEs.
- Prove the existence, uniqueness and regularity of the weak solution of the homogeneous Poisson BVP.

Week	Date	Sec.	Topics	Suggested Homework Problems
1	Jan 14 – 18		The linear first-order equation The significance of characteristics	
2	Jan 21 – 25		The Quasilinear equations Second order PDEs in two variables: classification	
3	Jan 28 - Feb 1		The hyperbolic canonical form The parabolic canonical form The elliptic canonical form	

4	Feb 4 – 8		The second-order Cauchy problem Characteristics and the Cauchy problem The wave equation : d'Alembert's solution of the the Cauchy problem d'Alembert solution as a sum of waves	
5	Feb 11– 15		The characteristic triangle The wave equation in 1-d A nonhomogeneous problem in 1-d	
6	Feb 18 – 21		A wave equation in 2-d The Kirchoff-Poisson solution of the wave equation in 3-d Hadamard's method of descent	
7	Feb 25- 29		The heat equation: IBVP The maximum principles	
8	March 3 – 7		The heat equation in 1-d The nonhomogeneous heat equation in 1-d The heat equation in 2-d	
<b>Midterm Exam</b>				
9	March 10- 14		Setting of Dirichlet and Neumann problems Some harmonic functions Representation theorems Maximum principle, Mean value property	
10	March 17- 21		Existence, Uniqueness and Well-posedness Dirichlet problem in 2-d Poisson's integral representation for a disk Green's function for a Dirichlet problem	
11-12	March 24- April 25		The Neumann problem in 2-d Sturm-Liouville theory Eigenvalue problem for the Laplace operator Application to solving non-homogeneous PDEs	
13-16	April 28– May 19		Introduction to Distribution & Sobolev spaces Weak solution of the Poisson BVP (existence, uniqueness and regularity) Energy methods to solve 2 <sup>nd</sup> order nonlinear PDEs	
<b>Final Exam</b>				

**Grading:**

Midterm Exam	35%
Homework assignments	20%
In class Presentations	10%
Final Exam	35%