KFUPM | Term 051 | Date: 26/11/2005
Mathematical Sciences | STAT 319 | Duration: 15 minutes
Quiz #3

Name: Solution Key ID#: Serial#: Section#: 5 6

Write neatly and eligibly

Q1 (6 points): A company modeled the lifetimes of the electromagnetic valve used for starting the idle-up actuator of an air conditioner by an exponential distribution with a rate of 0.05 failures per million revolutions. Consider a randomly selected electromagnetic valve.

a. Find the probability that this valve fails ***within*** the first ***half*** million revolutions.
b. Find the ***median*** time to failure for this valve.
c. Find the ***expected*** time to failure for this valve.
d. Find the ***standard deviation*** for the time to failure for this valve.

Let X: # of revolutions (in millions) to failure of a valve. ⇒

① $X: Exp(\frac{1}{0.05}) \Rightarrow X: Exp(20) \Rightarrow \boxed{\beta = 20}$

(a) $P(X \le x) = F(x) = 1 - e^{-x/\beta} \Rightarrow P(X \le \frac{1}{2}) = 1 - e^{-x/20}$ ①

$\Rightarrow P(X \le \frac{1}{2}) = F(\frac{1}{2}) = 1 - e^{-1/40} = \boxed{0.025}$ ①

(b) let median $= \tilde{x} \Rightarrow F(\tilde{x}) = P(X \le \tilde{x}) = 0.5$

① $\Rightarrow 1 - e^{-\tilde{x}/20} = 0.5 \Rightarrow 0.5 = e^{-\tilde{x}/20} \Rightarrow -\frac{\tilde{x}}{20} = \ln 0.5$

$\Rightarrow \tilde{x} = -20 \ln 0.5 = \boxed{13.863}$ million revolutions.

① (c) $E(X) = \mu_X = \beta = 20$ million revolutions.

① (d) $SD(X) = \sigma_X = \sqrt{\beta^2} = \beta = 20$

Q2 (4 points): The length of an injected-molded plastic case that holds magnetic tape is normally distributed with an average length of 90.2 mm and a standard deviation of 0.1 mm.

a. What is the probability that a part is ***longer*** than **90.3** mm?
b. If a ***total*** of **5%** of the shortest and the longest parts are scrapped, what are the ***minimum*** and ***maximum*** acceptable lengths of the injected-molded plastic case?

X: Length of an injected-molded plastic case in mm

$X: N(90.2, 0.01)$, $\mu = 90.2$, $\sigma = 0.1$ mm

(a) $P(X > 90.3) = P(Z > \frac{90.3 - 90.2}{0.1}) = P(Z > 1)$

$= P(Z < -1) = \boxed{0.1587}$ ①

① (b) $P(a \le X \le b) = 0.95 \Rightarrow$

$P(X \le a) = 0.025 \Rightarrow P(Z \le \frac{a - 90.2}{0.1}) = 0.025$

& $P(X \le b) = 0.975 \Rightarrow P(Z \le \frac{b - 90.2}{0.1}) = 0.025$ ①

$\Rightarrow \frac{a - 90.2}{0.1} = -1.96 \Rightarrow a = -0.196 + 90.2 = \boxed{90.004} = \min$

& $\frac{b - 90.2}{0.1} = 1.96 \Rightarrow b = 0.196 + 90.2 = \boxed{9.396} = \max$ ①

Q2 (Bonus 5 points): The manufacturing of semiconductor chips produces 2% defective chips. Assume the chips are independent and that a lot contains 1000 chips.

a. Find the ***exact*** probability that ***no*** chip is defective.
b. Approximate the probability that ***more*** than **25** chips are defective.
c. Approximate the ***third quartile*** of the number of defective chips.

$n = 1000$ (very large), $p = 0.02 \to 0 \Rightarrow$ The Poisson approximation to the Binomial CAN be used:

(1) @ Let X: # defective chips $\Rightarrow X : B(1000, 0.02)$

$$P(X=0) = \binom{1000}{0}(0.02)^0(0.98)^{1000} = 1.683 \times 10^{-9} \approx 0$$

(b) $P(X>25) = P(X \geq 26) \cong P(X \geq 25.5)$ (1)

$$\cong P\left(Z \geq \frac{25.5-20}{\sqrt{19.6}}\right) = P\left(Z \geq \frac{5.5}{4.427}\right)$$

$$= P(Z > 1.24) = P(Z < -1.24)$$

$$= \boxed{0.1075} \quad (1)$$

(c) $P(X < Q_3) = 0.75$ (1) $\Rightarrow P\left(Z < \frac{Q_3 - 20}{4.427}\right) = 0.75$

$$\Rightarrow \frac{Q_3 - 20}{4.427} = 0.67 \Rightarrow Q_3 = (0.67)(4.427) + 20$$

$$\Rightarrow \boxed{Q_3 = 22.966} \overset{(1)}{\approx} \boxed{23}$$

With My Best Wishes