King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program
Class test # 1

St. ID: _____ **Serial#:**____ **Section:**____ St. Name:_____

Q1: **Find the domain** for the following functions:

a) $f(x)=\dfrac{2-x}{2^x-1}$

Domain = $(-\infty,0)\cup(0,\infty)$

b) $f(x)=\sqrt{\dfrac{x}{1+e^x}}$

Domain = $(-\infty,\infty)$

c) $f(x)=\log(\log_3(x))$

Domain = $(1,\infty)$

Q2: **Find the arc length** that subtends a central angle of measure 60° in a circle of diameter 6 cm.

$\theta=60^\circ\dfrac{\pi}{180^\circ}=\dfrac{\pi}{3}$, $r=3$

$S=(3)\left(\dfrac{\pi}{3}\right)=\pi$

Q3: If the terminal side for the coterminal angle of θ pass through the point (-12, 5), **find** $\csc(\theta)+\cot(\theta)$?

Use the right triangle which has sides: -12, 5 and hypotenuse = 13.

$\therefore \csc(\theta)=\dfrac{13}{5}$ *and* $\cot(\theta)=-\dfrac{12}{5}$

$\Rightarrow \csc(\theta)+\cot(\theta)=\dfrac{13}{5}-\dfrac{12}{5}=\dfrac{1}{5}.$

Q4: **Solve**:

a) $\frac{e^{x}-e^{-x}}{2}=15 \Rightarrow e^{2x}-1=30e^{x} \Rightarrow \left(e^{x}\right)^{2}-30e^{x}-1=0$

$let\ u=e^{x} \Rightarrow u^{2}-30u-1=0$

$u=\frac{30\pm\sqrt{900+4}}{2} \Rightarrow e^{x}=\frac{30+\sqrt{900+4}}{2}$

$x=\ln\left[\frac{30+\sqrt{900+4}}{2}\right]$

b) $\log(2x^{2}+3x)=\log(10x+30)$

$\Rightarrow 2x^{2}+3x=10x+30$

$\Rightarrow 2x^{2}-7x-30=0$

$\Rightarrow (2x+5)(x-6)=0$ and both of them satisfy the equation.

$\Rightarrow x_{1}=6, x_{2}=-\frac{5}{2}.$

Q5: **Evaluate**:

a) $\ln(2-\ln e)+e^{3\ln(\sqrt[3]{2})}$

$=\ln(1)+\left(\sqrt[3]{2}\right)^{3}=1$

b) $(\log_{5}40-1).\log_{8}5$

$=\log_{5}(40)\log_{8}(5)-\log_{8}(5)$

$=\log_{8}(40)-\log_{8}(5)=\log_{8}(\frac{40}{5})=1$

c) $\sin(255^{\circ})=-\sin(75)=-\sin(30+45)$

$=-[\sin(30)\cos(45)+\sin(45)\cos(30)$

$=-\left[\frac{\sqrt{6}+\sqrt{2}}{4}\right]$

d) (x,y) Such that $W\left(-\frac{31\pi}{6}\right)=(x,y)$, where W is the wrapping function.

$x=\cos(-\frac{31\pi}{6})=-\cos(\frac{\pi}{6})=-\frac{\sqrt{3}}{2}$

$y=\sin(-\frac{31\pi}{6})=\sin(\frac{\pi}{6})=\frac{1}{2}$

$(x,y)=(-\frac{\sqrt{3}}{2},\frac{1}{2})$

$-\frac{31}{6}\pi=\alpha+2\pi k \quad , \quad k=-3$

$\alpha=\frac{5}{6}\pi \Rightarrow \theta'=\frac{\pi}{6}.\ (\theta \ \ in\ quadrant\ II)$

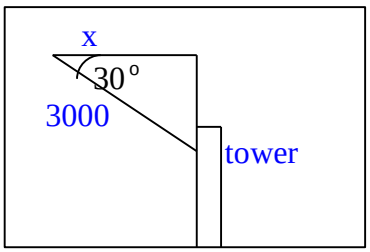
Q6: **Verify** the following identities:

a) $\dfrac{\cos(x)\tan(x)+2\cos(x)-\tan(x)-2}{\tan(x)+2}=\cos(x)-1$

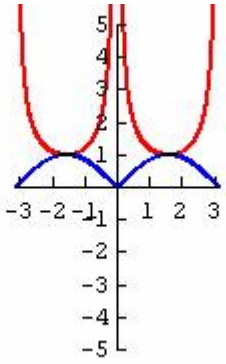
b) $\sqrt{\dfrac{1+\sin(x)}{1-\sin(x)}}=\dfrac{1+\sin(x)}{\cos(x)}$, $\cos(x)>0$.

Q7: If the distance from a helicopter to a tower is 3000 feet and the angle of depression is 30°, **find** the distance on the ground from a point directly below the helicopter to the tower.

$\cos(30^{\circ})=\dfrac{x}{3000}\Rightarrow x=3000\dfrac{\sqrt{3}}{2}=1500\sqrt{3}$.



Q8: **Sketch** the graph for $y=|\csc(x)|$ on the interval $[-\pi,\pi]$



Q9: Consider the function $y = 2\cot\left(3x - \frac{\pi}{2}\right)$, $x \in \left(-\frac{\pi}{4}, \pi\right)$,

a) The phase shift.

$\frac{\pi}{6}$

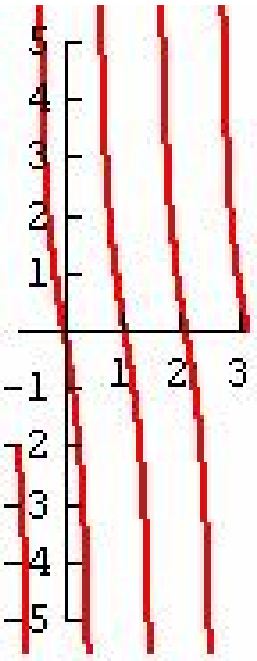
b) The intercepts.

$$x = 0, \frac{\pi}{3}, 2\frac{\pi}{3}.$$

c) The vertical asymptotes.

$$x = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, 5\frac{\pi}{6}.$$

d) Sketch the graph for y.



Good Luck