Sec: 6.2	Page: 484	Problem: 5

Find the exact value of $\tan(45^\circ - 30^\circ)$.

Solution:

$$\tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{\left(3-\sqrt{3}\right)\left(3-\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)} = \frac{9-6\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$$

Written by: S. Omar

Sec: 6.2	Page: 484	Problem: 10

Find the exact value of $\sin\frac{5\pi}{12}\cos\frac{\pi}{4} - \cos\frac{5\pi}{12}\sin\frac{\pi}{4}$.

Solution:

$$\sin\frac{5\pi}{12}\cos\frac{\pi}{4} - \cos\frac{5\pi}{12}\sin\frac{\pi}{4} = \sin\left(\frac{5\pi}{12} - \frac{\pi}{4}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

Written by: S. Omar

Sec: 6.2	Page: 484	Problem: 15

Find the exact value of $\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$.

Solution:

$$\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

Written by: S. Omar

Sec: 6.2	Page: 484	Problem: 22

Write the expression $\cos 4x \cos 2x - \sin 4x \sin 2x$ in terms of single trigonometric function.

Solution:

$$\cos 4x \cos 2x - \sin 4x \sin 2x = \cos(4x + 2x) = \cos 6x$$

Written by: S. Omar

Sec: 6.2	Page: 484	Problem: 29

Write the expression $\dfrac{\tan 3x + \tan 4x}{1 - \tan 3x \tan 4x}$ in terms of single trigonometric function.

Solution:

$$\frac{\tan 3x + \tan 4x}{1 - \tan 3x \tan 4x} = \tan(3x + 4x) = \tan 7x$$

Written by: S. Omar

Sec: 6.2	Page: 484	Problem: 33

Given $\sin\alpha = 3/5$, α in Quadrant I, and $\cos \ \ = -5/13$, in Quadrant II, find

(a): $\sin(\alpha - \ \ )$ **(b):** $\cos(\alpha + \ \ )$ **(c):** $\tan(\alpha - \ \ )$

Solution:

$$\sin\alpha = \frac{3}{5} \overset{\alpha \text{ is in QI}}{\Rightarrow} \cos\alpha = +\sqrt{1 - \sin^2\alpha} = +\sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\cos \ \ = -\frac{5}{13} \overset{\text{is in QII}}{\Rightarrow} \sin \ \ = +\sqrt{1 - \cos^2 \ \ } = +\sqrt{1 - \frac{25}{13^2}} = \frac{12}{13}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\tan \ \ = \frac{\sin \ \ }{\cos \ \ } = \frac{12/13}{-5/13} = -\frac{12}{5}$$

(a): $\sin(\alpha - \ \ ) = \sin\alpha \cos \ \ - \cos\alpha \sin \ \ $

$$= \frac{3}{5}\left(-\frac{5}{13}\right) - \frac{4}{5}\left(\frac{12}{13}\right) = -\frac{15}{65} - \frac{48}{65} = -\frac{63}{65}$$

(b): $\cos(\alpha + \ \ ) = \cos\alpha \cos \ \ - \sin\alpha \sin \ \ $

$$= \frac{4}{5}\left(-\frac{5}{13}\right) - \frac{3}{5} \cdot \frac{12}{13} = -\frac{20}{65} - \frac{36}{65} = -\frac{56}{65}$$

(c): $\tan(\alpha - \ ) = \dfrac{\tan\alpha - \tan}{1+\tan\alpha\tan}$

$$= \dfrac{\frac{3}{4} - \left(-\frac{12}{5}\right)}{1+\left(\frac{3}{4}\right)\left(-\frac{12}{5}\right)} = \dfrac{\frac{15+48}{20}}{\frac{20-36}{20}} = -\frac{63}{16}$$

Written by: S. Omar

Sec: 6.2	Page: 485	**Problem: 40**

Given $\cos\alpha = 8/17$, α in Quadrant IV, and $\sin \ = -24/25$, in Quadrant III, find

(a): $\sin(\alpha - \ )$ **(b):** $\cos(\alpha + \ )$ **(c):** $\tan(\alpha + \ )$

Solution:

$$\cos\alpha = 8/17 \overset{\alpha \text{ is in QIV}}{\Rightarrow} \sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{64}{17^2}} = -\frac{15}{17}$$

$$\sin \ = -24/25 \overset{\text{is in QIII}}{\Rightarrow} \cos \ = -\sqrt{1-\sin^2} = -\sqrt{1-\frac{24^2}{25^2}} = -\sqrt{\frac{25^2-24^2}{25^2}} = -\frac{\sqrt{(24+1)^2-24^2}}{25} = -\frac{7}{25}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-15/17}{8/17} = -\frac{15}{8}$$

$$\tan \ = \frac{\sin}{\cos} = \frac{-24/25}{-7/25} = \frac{24}{7}$$

(a): $\sin(\alpha - \ ) = \sin\alpha\cos \ - \cos\alpha\sin$

$$= \left(-\frac{15}{17}\right)\left(-\frac{7}{25}\right) - \frac{8}{17}\left(-\frac{24}{25}\right) = \frac{105}{425} + \frac{192}{425} = \frac{297}{425}$$

(b): $\cos(\alpha + \ ) = \cos\alpha\cos \ - \sin\alpha\sin$

$$= \frac{8}{17}\left(-\frac{7}{25}\right) - \left(-\frac{15}{17}\right)\left(-\frac{24}{25}\right) = -\frac{56}{425} - \frac{360}{425} = -\frac{416}{425}$$

(c): $\tan(\alpha + \ ) = \dfrac{\tan\alpha + \tan}{1-\tan\alpha\tan}$

$$= \dfrac{-\frac{15}{8}+\frac{24}{7}}{1-\left(-\frac{15}{8}\right)\left(\frac{24}{7}\right)} = \dfrac{\frac{-105+192}{56}}{1+\frac{360}{56}} = \frac{87}{416}$$

Written by: S. Omar

Sec: 6.2	Page: 485	Problem: 67

Write the expression $\cos(\theta + 3\pi)$ as a function than involve only $\cos\theta$.

Solution:

$$\cos(\theta + 3\pi) = \cos\theta \cos 3\pi - \sin\theta \sin 3\pi$$
$$= \cos\theta(-1) - \sin\theta(0)$$
$$= -\cos\theta$$

Written by: S. Omar

Sec: 6.2	Page: 485	Problem: 80

Verify the identity $\dfrac{\sin(x+y)}{\sin x \sin y} = \cot x + \cot y$.

Solution:

$$\text{LHS} = \frac{\sin(x+y)}{\sin x \sin y} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \sin y}$$
$$= \frac{\sin x \cos y}{\sin x \sin y} + \frac{\cos x \sin y}{\sin x \sin y}$$
$$= \frac{\cos y}{\sin y} + \frac{\cos x}{\sin x}$$
$$= \cot y + \cot x = \text{RHS}$$

Written by: S. Omar