

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics - STAT-319-Term071-Quiz4

Name: _____

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Serial: _____

Q1. Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

a. What is the probability that your first call that connects is your tenth call?

$$p = 0.02, q = 1 - p = 1 - 0.02 = 0.98 \quad X \sim \text{Geometric}(p = 0.02)$$

$$P(X = 10) = (0.02)(0.98)^{10-1} = 0.0167$$

b. What is the probability that a man connect twice in a sample of size 5 calls?

$$n = 5, p = 0.02, q = 1 - p = 1 - 0.02 = 0.98 \quad X \sim \text{Binomial}(n = 5, p = 0.02)$$

$$P(X = 2) = \binom{5}{2} (0.02)^2 (0.98)^3 = 0.0038$$

Q2. The line width of for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

a. What is the probability that a line width is greater than 0.62 micrometer?

$$\begin{aligned} P(X > 0.62) &= P\left(\frac{X - 0.5}{0.05} > \frac{0.62 - 0.5}{0.05}\right) = P(Z > 2.4) \\ &= 1 - P(Z < 2.4) = 1 - 0.9918 = 0.0082 \end{aligned}$$

b. The line width of 90% of samples is below what value?(Hint: find the 90th percentile)

$$\begin{aligned} \text{Let the value be } k: P(X < k) = 0.9 &\Rightarrow P\left(\frac{X - 0.5}{0.05} < \frac{k - 0.5}{0.05}\right) = P\left(Z < \frac{k - 0.5}{0.05}\right) \\ &\Rightarrow \frac{k - 0.5}{0.05} = 1.28 \Rightarrow k = (0.05)(1.28) + 0.5 = 0.564 \end{aligned}$$

Q3 The time between arrivals of taxis at a busy intersection is exponentially distributed with a mean of 10 minutes.

a. What is the probability that you wait longer than one hour for a taxi?

$$\mu = \beta = 10 \Rightarrow f(x) = \begin{cases} \frac{1}{10} e^{-\frac{x}{10}}, & x > 0 \\ 0 & , \text{ elsewhere} \end{cases}$$

$$P(X > 1 \text{ hour}) = P(X > 60 \text{ minutes}) = \int_{60}^{\infty} \frac{1}{10} e^{-\frac{x}{10}} dx = -e^{-\frac{x}{10}} \Big|_{60}^{\infty} = e^{-6} = 0.0025$$

b. What is the probability that at least two of the next 5 persons will wait longer than one hour for a taxi?

$$X \sim \text{Binomial}(n = 5, p = P(X > 1 \text{ hour}) = P(X > 60 \text{ minutes}) = 0.0025)$$

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) = 1 - \left[\binom{5}{0} (0.0025)^0 (0.9975)^5 + \binom{5}{1} (0.0025)^1 (0.9975)^4 \right] \\ &= 1 - [0.98756 + 0.012375] = 1 - 0.999935 = 0.000065 \end{aligned}$$