Quiz3-SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics - STAT-319-Term071

Name: ID: Sec.: Serial:

Q1. The number of messages sent per hour over a computer network has the following distribution:
x is the number of messages

X	10	11	12	13	14	15
f(x)	0.08	0.15	0.30	0.20	0.20	0.07

Determine the following

a. Mean of the number of messages sent per hour.

$$\mu = E(X) = \sum x\, f(x) = (10)(0.08)+(11)(0.15)+(12)(0.3)+(13)(0.2)+(14)(0.2)+(15)(0.07)$$
$$= 0.8 + 1.65 + 3.60 + 2.60 + 2.80 + 1.05 + = 12.05$$

(2-Points)

b. Standard deviation of the number of messages sent per hour.

$$\sigma = \sqrt{E(X^2) - \mu^2}$$
$$E(X^2) = \sum x^2 f(x)$$
$$= (10)^2(0.08)+(11)^2(0.15)+(12)^2(0.3)+(13)^2(0.2)+(14)^2(0.2)+(15)^2(0.07)$$
$$= 8 + 18.15 + 43.2 + 33.8 + 39.2 + 15.75 = 158.1$$
$$\Rightarrow \quad \sigma = \sqrt{158.1 - (12.5)^2} = \sqrt{1.85} = 1.3601$$

(3-Points)

Q2. The thickness of a conductive coating in micrometers has a density function given by

$$f(x) = \begin{cases} \dfrac{600}{x^2} & , \ 100 < x < 120 \\ 0 & , \ elsewhere \end{cases}$$

a. What is the probability that the thickness of a certain conductive coating more than 112

$$P(X > 12) = \int_{112}^{120} \frac{600}{x^2} dx = -\frac{600}{x}\Big|_{112}^{120} = -\left(5 - \frac{75}{14}\right) = \frac{5}{14} = 0.3571$$

(2-Points)

b. If $Y = (X^3 + 3X^2 - 2)$, find the expected value of Y.

$$E(Y) = E(X^3 + 3X^3 - 2) = E(X^3) + 3E(X^2) - 2$$
$$E(X^3) = \int_{100}^{120} x^3 . \frac{600}{x^2} dx = \int_{100}^{120} 600x\, dx = 300x^2\Big|_{100}^{120} = 4,320,000 - 3,000,000 = 1,320,000$$
$$E(X^2) = \int_{100}^{120} x^2 . \frac{600}{x^2} dx = \int_{100}^{120} 600\, dx = 600(120 - 100) = 12,000$$
$$E(Y) = 1,320,000 + 3(12,000) - 2 = 1,355,998$$

(3-Points)